Central Bank Asset Purchases and Lending: Impact on Search Frictions

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Abstract

Central banks implement a combination of two monetary policies: asset purchases and asset lending. The primary purpose of asset purchases is to lower interest rates; however, this impairs bond market liquidity, so central banks supplement the function of market liquidity by lending purchased assets. We apply a search-theoretic model to explore the impact of the securities lending facility (SLF) of central banks by introducing a central bank as a lender as well as a purchaser. We test three model predictions using intraday data from a Japanese government bond electronic platform. First, we find large-scale asset purchases (LSAPs) increase order imbalance in the repo market. Threshold analysis reveals that asset purchase amounts exceeding ¥9 billion (equivalent to 1.4% of the outstanding) generate a significantly higher proportion of bid orders. Second, the SLF rate has a ceiling effect on the repo rate by affecting dealers' choice between the repo market and the SLF. Third, new concrete friction measures are tested, showing that LSAPs and the SLF have opposite influences on bargaining power in the repo market.

Keywords: Order imbalance, sovereign bonds, large-scale asset purchase, bargaining power, repo

JEL classification: C54; E43; E52; E58; G12; G14

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1 Introduction

Since the global financial crisis in 2008, the world's major central banks have engaged in unconventional monetary policies in the form of quantitative easing (QE) programs to stimulate their economies. These policies, however, have commonly reduced the availability of bonds from the secondary market, impairing bond market liquidity and increasing dealers' difficulty in covering short positions. Dealers who make the market for government bonds engage in short covering either to buy bonds in the secondary market or to borrow them through the repurchase agreement (repo) market. When a bond is scarce in the market, dealers sometimes cannot fill the quantity of shares required. This study focuses on such market situations, in which counterparties become harder to find because a large fraction of qualified owners have been reduced by the aggressive large-scale asset purchases (LSAPs) of a central bank. The number of failed transactions in the Japanese government bond (JGB) market increased by 2.36 times in the six years since 2013, when the Bank of Japan (BoJ) introduced aggressive LSAPs, and the amount of failed transactions increased by 2.28 times, from ¥2.98 trillion in 2013 to ¥6.81 trillion in 2019, according to statistics published by the BoJ. The BoJ statistics indicate increased search frictions.

The frequency of failures increases when it becomes more difficult to locate a counterparty who is willing to trade a particular security. According to Amihud et al. (2005), this is another source of illiquidity. A borrower (short seller) must negotiate the price with the counterparty in a less than perfectly competitive environment, since alternative trading partners are not immediately available. This search friction is particularly relevant in over-the-counter (OTC) markets in which there is no central marketplace. Corradin and Maddaloni (2020) show the importance of security-specific demand, liquidity, and the link to short selling activity. The relation between scarcity and liquidity in the government bond markets is analyzed by **?**, Musto et al. (2018), Pelizzon et al. (2018), Corradin and Maddaloni (2020), and Ferdinandusse et al. (2020).

Facing increased search friction, central banks in the US, Europe and Japan have strengthened their securities lending facilities (SLFs) to mitigate bond supply shortages. In Japan, the BoJ facilitates a short-term SLF to maintain the liquidity of the underlying market. The SLF can be a last resort for short sellers to cover their position; therefore, a central bank can act as a competitor of the repo market. The more aggressively a central bank implements LSAPs, the higher the demand for lending through the SLF because LSAP reduces availability of bonds in repo market (Kinugasa and Nagano (2017); D'Amico et al. (2018)). This paper investigates the relation between LSAPs and the SLF, and these impact on search frictions.

Duffie et al. (2002) has developed a search-based theory for the OTC securities lending market. Their model implies that the lending fee effects are greater for a smaller float. Vayanos and Weill (2008) have extended that work to a multiple-asset model to clarify the premium of on-the-run bonds. In their calibration model, borrowers' longer search times (or duration times) are associated with greater specialness in the repo market, due to the lower competition between lenders. Corradin and Maddaloni (2020) extend the model of Vayanos and Weill

(2008) by introducing the central bank as the buy-and-hold investor and use search times to explore cash liquidity as well as repo liquidity in normal and crisis periods. Ferdinandusse et al. (2020) show that it becomes harder for bond buyers to find sellers as the stock of bonds becomes depleted on the secondary market by the QE program.

We build a search-theoretic model introducing a central bank as a lender in the government bond market. Our model differs from that of Corradin and Maddaloni (2020) in that they assume the central bank is a buy-and-hold investor, whereas we consider the central bank as a lender as well.[1] Our model considers the role of the SLF as an alternative for a dealer's short covering activity which is not analyzed by preceding literature. We model interactions between borrowers, lenders, and a central bank and solve the utility functions for the repo lending fee through Nash bargaining in the presence of a central bank. We calibrate the model using parameters derived from actual data from the JGB market before and after the relaxation of the SLF conditions in June 2019, when the BoJ changed the lending rate. Our model predicts that the SLF works to keep repo lending fees from indefinitely rising and has a ceiling effect on repo lending fees. The ceiling level is determined by the SLF rate setting. LSAPs decrease the repo supply (amounts held by non–central bank investors), resulting higher lending fees in repo market. The central bank's lending mitigates this increase and influence trading activity in the repo market. The model calibration also predicts that borrowers will spend more time to find a counterparty, given greater scarcity in the bond market.

In this paper, we examine a central bank's combination of two policies: LSAPs and the SLF. A central bank can decide the amount of LSAPs and its lending rate to influence the role of the repo market. The SLF has effects of countercyclical policy on the liquidity of the repo market. The government bond market in Japan provides an ideal setting for examining the trade-off between asset purchases and lending. No study, as far as we know, has both theoretically and empirically investigated the policy trade-off between a central bank's LSAPs and SLF. In our sample period, from April 2016 to December 2019, the BoJ implemented LSAP programs by targeting JGBs. The BoJ's holding of nominal JGBs reached ¥453 trillion in December 2019, corresponding to about 81.3% of Japan's nominal gross domestic product, while its average holding ratio across nominal JGBs rose to 46.3%, unprecedented in the annals of central bank history. The BoJ holdings of some JGBs exceeded 85% of the outstanding debt, providing a natural experiment to determine how scarcity affects repo rate and search frictions across bonds and maturities. In parallel with the LSAPs, the BoJ lent securities from its holdings through the SLF. During our sample period, the BoJ relaxed the conditions for SLF lending. We use this event to investigate the impact of the SLF lending rate on repo transactions.

We rely on order submission data from the JBond Totan Securities (JBOND) repo platform (e-platform), which provides functions similar to a limit order market.[2] The platform's

[1]In their model, the central bank does not have a function of the SLF because the ECB did not lend out the securities purchased through the Securities Markets Program (SMP). Their model also differs from ours in that they model both outright and repo transactions.

[2]Kinugasa and Nagano (2017) uses the data from the same platform. But their empirical analyses don't use intraday order submission data.

structure differs from the search-theoretic model's assumption of an OTC market. However, the activities observed through the platform reflect the search process of individual traders in the OTC market. Traders typically submit a bid order with a favorable price to the e-platform while they search for the same bond in the OTC market and negotiate a price at which they can borrow. Activities in the OTC market are not observable, but the quoting behavior and execution status on the e-platform reflect the difficulty of traders' search activity in the OTC market. If a trader finds a counterparty for the shorting bond, the trader immediately cancels the order submitted to the e-platform to avoid double execution. The trader can change bid quotes to expedite the order's execution. Through the platform's quoting behavior, we can calculate measures that provide more direct evidence of the search process than those provided by previous studies. This is our unique contribution in the empirical study for search frictions. We measure the interval between the initial submission time and the time of execution or cancellation and the associated difference between the initial bid rate and the filled (or final) rate. We expect greater scarcity to be associated with higher costs and longer order duration times.

In our empirical studies, we focus on five- and 10-year government bonds and amounts outstanding that are greater and a BoJ holding share that is higher than in other segments. First, we examine the influence of the central bank's purchase operations in the form of auctions on dealer activity in the repo market. If an auction bidder does not hold the bond to be sold to the central bank, the bidder is more likely to locate the bond in the repo market, since the particular bond is already scarce. Thus, central bank purchase operations must increase repo bid orders. We expect the greater the scarcity, the larger the demand for the repo transaction. We look at the order imbalance, which is calculated as the proportion of bid orders among all orders. Our empirical results show that bid orders outnumber offer orders for the bonds, of which the BoJ holds 60%, and the impact on order imbalance grows as the purchased amounts in a single operation increase. The results of threshold dummy variable analysis indicate that the sensitivity of the order imbalance is positively different when the amounts purchased by the BoJ exceed ¥9 billion, equivalent to 1.4% of the outstanding. The results suggest that the central bank should take care to ensure that a purchase operation does not have too great of an impact on the repo transaction when implementing LSAPs.

Second, we investigate the interaction between the central bank's SLF and the repo transaction. Since the central bank's lending operations are implemented on the settlement day of the repo transaction, traders in the repo market can place orders while considering the predicted SLF lending rate. A borrower places a bid order to the repo market and tries to find a counterparty who is willing to trade at a better rate than that from the SLF. Therefore, the repo rate has a ceiling imposed by the SLF. More specifically, borrowers' activities to cover short positions are expected to depend on the rate difference between the repo market and the SLF. We test this by panel regression analysis, and the results are consistent with our expectation. Previous empirical literature reports the central bank's lending reduces specialness (Arrata et al. (2020); Kinugasa and Nagano (2017)). In contrast, our calibration exercise and empirical analyses show the ceiling effect on the repo rate.

We test whether a change in the ceiling is observed along with the relaxation, using the event in June 2019 when the BoJ relaxed the conditions for the SLF. Our empirical analyses show that the rise of the SLF lending rate from −50 bps to −35 bps increases the proportion of transactions at a higher rate in the repo market. We find the ceiling level changes according to the SLF lending rate, which indicates market participants build their rate expectations based on the level of the SLF rate. The SLF lending rate set by the central bank has a great influence on borrowers' behavior. This also means that when the SLF rate is lower than the equilibrium rate determined by true supply/demand situation, proportion of lending through the SLF should increase. After the rise in the SLF rate, although the total amount of lending through the SLF do not increase significantly, the proportion for SLF lending increases for bonds whose repo rate increases above the SLF rate. The revision of the SLF rate setting does not have a distorting effect on trading activities. The panel probit regression shows the likelihood of SLF lending depends on the rate difference between the repo market and the SLF.

Third, we investigate the search friction by using two new measures, namely, the duration time until execution and the rate concession amount. Given greater scarcity, a dealer with a short position will be forced to spend longer locating a specific bond and to pay higher prices than the initial price to avoid failure to deliver. As in Vayanos and Weill (2008), repo liquidity can be measured by duration times in the search-theoretic model. We measure actual duration times by order submission data and test their model prediction. As for the first search friction measure, duration time, our empirical analyses show that the higher the holding ratio, the longer the duration until execution. When the holding ratio is below 20%, the duration time is 7.96 minutes, and when it rises above 80%, the duration becomes 17.89 minutes. When the holding ratio is above 40%, the duration time increases steadily, which indicates that bond scarcity affects the duration time of the order execution. Another friction measure is the rate concession amount made by bidders or offerers. The average rate of the concession amounts is greater for scarcer bonds. The concession amounts for bid orders are 0.117–0.143 bps for bonds less than 60% of the BoJ's holdings, increasing to 0.351 for those with 60–80% of the BoJ's holdings and to 0.401 bps for those with over 80% of the BoJ's holdings.

A novel result is also provided that the SLF rate setting changes the bargaining power of lenders and borrowers in the repo market. We focus on the relative rate concession of lenders and borrowers. We expect lenders (borrowers) with greater bargaining power to offer lower concessions. We consider the ratio of the rate concession amounts for offer orders to the aggregate rate concession amounts to be an indicator of the bargaining power of borrowers in the search-theoretic models applied by Duffie et al. (2002), Duffie et al. (2007), and many others including us. Before the relaxation period, this indicator is calculated at 0.530 for bonds with less than 20% of the holdings and declines to 0.438–0.447 for bonds with 40–80% of the holdings. This result indicates the aggressive stance of the offer side and that the bargaining power of lenders is stronger than that of borrowers, reflecting the increasing scarcity of government bonds. On the other hand, after the relaxation, the bargaining power of borrowers increases, which indicates the relaxation of the SLF conditions changes the relative bargaining power, such that lenders need to concede greater rates for execution.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 describes the repo market, the LSAP program, and the SLF conducted by the BoJ. Section 4 constructs a model based on search theory and presents the research hypotheses. Section 5 describes the empirical methodology, and Section 6 presents the empirical results. Section 7 concludes the paper.

2 Literature

The seminal work of Duffie (1996) specifies a model describing the link between the repo and cash markets and shows that bonds trading on specialness should carry a price premium in the cash market. Jordan and Jordan (1997) empirically test most of Duffie's predictions and shed light on the role played by the liquidity of bonds (on-the-run issues) and the holders of a security, introducing the concept of the availability of a specific security. Both Duffie (1996) and Jordan and Jordan (1997) use sample data from the US repo market.

Duffie et al. (2002) have developed a search-based theory for the securities lending market, extended to a multiple-asset model by Vayanos and Weill (2008). Duffie et al. (2002) study the OTC market and construct a dynamic model for the determination of prices, lending fees, and short interest (the quantity of securities held short). Their model implies that lending fee effects are greater for a smaller float. The expected price decline associated with lending fees is then likely to be more pronounced in situations characterized by a high degree of belief heterogeneity and a small number of circulating shares. In the calibration model proposed by Vayanos and Weill (2008), borrowers' longer search times are associated with greater specialness in the repo market, due to the lower competition between lenders. The authors aim to clarify the premium of on-the-run bonds, but do not consider scarcity. Corradin and Maddaloni (2020), whose paper is one of the closest to ours, extend the model of Vayanos and Weill (2008) by introducing the central bank as a buy-and-hold investor and explore the impact of the central bank's purchases on the scarcity premiums in the repo market during the sovereign debt crisis. The study of Corradin and Maddaloni (2020) and ours are similar in that, based on a search-theoretic model, they both address the case in which the demand for repo transactions is high because of the purchase operations of the central bank and collateral bonds have become scarce in the market due to LSAPs. However, unlike their setting, in our model the central bank acts not only as a buy-and-hold investor, but also as a lender through its lending facility, which allows us to analyze dealers' choice between repo market and the SLF. Liu and Wu (2016) also extend the model of Vayanos and Weill (2008) to account for the effects of counterparty default risk in the repo market on the lending fee and the pricing of Treasury securities. Liu and Wu (2016) consider that counterparty risk reduces lenders' or borrowers' willingness to supply funds and collateral and incentives to short-sell and lend. They show that on-/off-the-run spreads are low when counterparty risk is high and that this relation was much stronger during the financial crisis.

Duffie et al. (2005) build a dynamic asset-pricing model that captures search and bargaining features and analytically derive equilibrium allocations, prices negotiated between investors, as well as market makers' bid and ask prices. Duffie et al. (2007) show that illiquidity discounts are higher when counterparties are harder to find or the fraction of qualified owners is smaller. Ferdinandusse et al. (2020) model sovereign bond markets with a search-theoretic framework based on that of Duffie et al. (2005). They show that, as the stock of bonds becomes depleted on the secondary market by the QE program, it becomes harder for buyers to find a seller. The authors predict that the QE program crowds out buyers besides the central bank and leads to lower bond liquidity.

There are many empirical works on the impacts of QE programs on the repo market. Among them, the study of D'Amico et al. (2018) quantifies the scarcity value of Treasury collateral by estimating the impact of security-specific demand and supply factors on the repo rates of all outstanding US Treasury securities. This scarcity effect seems to pass through to Treasury cash market prices, providing additional evidence of the scarcity channel of QE. The US Federal Reserve System's reverse repo operations could help reduce scarcity premiums by alleviating potential shortages of high-quality collateral. Kinugasa and Nagano (2017) examine the impact of the BoJ's quantitative and qualitative monetary easing (QQE) on repo specialness, using repo transaction data from May 2014 to March 2017. They show that the BoJ's holding ratio of JGBs increases repo specialness and the BoJ's SLF mitigates bond scarcity. Song and Zhu (2018) investigate the Fed's purchases of the mortgage-backed-securities (MBS) and show the determinants of dollar roll specialness: how much implied financing rates fall below MBS repo rates.

The works of both Corradin and Maddaloni (2020) and Musto et al. (2018) are related to our empirical approach in terms of investigating the link between short selling activities and specialness. Corradin and Maddaloni (2020) highlight the importance of security-specific demand and analyze the determinants of the quantiles of the distribution of specialness. They show that very special bonds are more sensitive to sizable changes in supply and demand. They also find the probability of failure to deliver increases with the specialness of the bond. Skinner and Dufour (2006) also analyze the Italian BTP repo market. Musto et al. (2018) show that a decline in the frequency of special trades is associated with an increase in the volume of failures. Local supply effects arising from the ECB's Securities Markets Programme purchases also had a similar impact, but they were mitigated by the introduction of penalties for failure to deliver. Dunne et al. (2011) analyze how the crisis affected the bidding behavior of banks in refinancing operations in the euro area. Mancini et al. (2016) conduct a comprehensive study of the European repo market and show that the importance of the central counterparty-based segment in this market makes it more resilient during crises and even acts as a shock absorber.

Boissel et al. (2017) argue that central clearing counterparties provide some protection in periods of intermediate sovereign stress (2009–2010), but this protection became ineffective at the peak of the sovereign crisis (in 2011). Buraschi and Menini (2002) analyze more specifically the relation between the current term structure of special repos and future collateral values, using data on the German government repo market. Arrata et al. (2020) show that most short-term interest rates in the euro area are below the ECB deposit facility rate, the rate at which the central bank remunerates banks for excess reserves. Using proprietary data from a public sector purchase program's purchases and repo transactions for specific (special) securities, the authors assess the scarcity channel of the public sector purchase program and its impact on repo rates.

The SLF run by the central bank has a competitive and complementary relation with the repo market. Papers on government bond lending by central banks are scarce and include those of Fleming (2002), Fleming et al. (2010), Kinugasa and Nagano (2017), and Arrata et al. (2020). Fleming (2002) studies the temporary supply of government bonds by the US Federal Reserve during periods of financial market turmoil. During the turmoil in the Treasury stock market in 2001, the Fed eased two lending requirements: reducing lending fees to two-thirds and easing the maximum lending amount, but no significant reduction in failure amounts was seen. The effect of mitigating confusion was weak.

Fleming et al. (2010) assess the effectiveness of the term SLF of the central bank and find that it significantly narrows repo spreads between Treasury collateral and less liquid collateral. The authors find that the effects are driven by operations in which appreciably less liquid securities can be pledged as collateral and that such operations increase the repo rates for liquid non-Treasury collateral. Kinugasa and Nagano (2017) include a dummy for SLF lending in their specialness regression model and show the BoJ's SLF narrows specialness. They also point out that the repo rate is priced with an awareness of the SLF lending rate. Arrata et al. (2020) study the interaction of the ECB's Public Sector Purchase Program with government bond repo rates and the impact of scarcity caused by the Public Sector Purchase Program. They show that the special collateral (SC) rate drops by 0.78 bps when 1% of the outstanding amount of government bonds is purchased. They also investigate the impact of the SLF run by the ECB in line with the theoretical framework built by Duffie and Krishnamurthy (2016) by using dummy variables after the implementation of SLF and show that the SLF has the effect of mitigating the decline in the SC rate.

3 Repo transactions, LSAPs, and SLF in Japan

In this section, we first describe the role of repo transactions in the JGB market and summarize recent LSAP programs in Japan, which have increased scarcity. We next explain the central bank's SLF, which has a competitive relation with repo transaction.

3.1 Repo transactions and a summary of recent purchase programs

A repo is a form of short-term (usually overnight) borrowing or lending in government securities. In a repo transaction, a lender turns over an asset to a borrower in exchange for cash. At maturity, the borrower returns the asset and the lender returns the cash, together with some previously agreed upon interest rate payment, called the repo rate.

The cash and repo markets are closely linked through short sales. The repo market is often used when a dealer creates short selling positions in the cash market. The most typical scenario is selling a bond short in the cash market while simultaneously borrowing the same bond through a repo to cover a short position. Many studies, such as Duffie's (1996), show that specialness increases with the amount of short selling activity in the cash market and is driven by the demand for short positions, constraints on the available supply, and the liquidity of the security.

The reduction in available bonds increases the likelihood of dealers being unable to close their short positions in the JGB cash market (Pelizzon et al. (2018)). Under market conditions with scarce bonds, the demand for procuring bonds in the repo market increases. The reduction in available bonds in the cash market is equivalent to the reduction of collateral for the repo market. The reduction of collateral increases the difficulty of borrowing the bonds. Therefore, the repo rate of a scarce bond should be at a lower level and its degree of specialness should be higher (e.g., Corradin and Maddaloni (2020), D'Amico et al. (2018), Arrata et al. (2020), Brand et al. (2019)).

In addition to the scarcity effect of LSAPs on repo rate, we also take into account the lending demand caused by each purchase operation. The BoJ implements purchase operations in the form of an auction. If an auction bidder does not hold a bond to be sold to the BoJ, the bidder will locate the bond in the repo market. We consider LSAPs by a central bank to have two different effects on the repo market: one is to increase the demand for repo transactions to cover short positions created in response to the central bank's purchase operations, and the other is to reduce the supply of repo collateral generated by the central bank's cumulative purchases. Both effects induce specialness.

In Japan, a purchase program had already begun before 2013, but, on April 4, 2013, the BoJ introduced QQE, which increased its purchases of JGBs to an annual amount of about ¥50 trillion.[3] On October 31, 2014, the BoJ announced the expansion of the QQE such that the purchase amount would increase at an annual pace of about ¥80 trillion, thus aiming to decrease interest rates across the entire yield curve and to shift its purchases further toward longer-term bonds. On January 29, 2016, the BoJ introduced QQE with a negative interest rate and revealed a policy of targeting negative interest rates and of continuing to purchase JGBs in amounts increasing by about ¥80 trillion annually. On September 21, 2016, the BoJ introduced QQE with yield curve control and announced its intent to purchase JGBs to maintain the 10-year JGB yield around 0%.[4]

Our sample period is from April 2016 to December 2019, which is about three years after the introduction of the aggressive QQE program, and the BoJ's holding ratio is thus high throughout our sample period. However, the amounts purchased by the BoJ have been changing.

[Table 1 about here.]

Panel A of Table 1 shows the monthly amounts (in billions of yen) of five- and 10-year JGBs purchased by the BoJ and its holding ratio (%) averaged over each year. The BoJ

[3]Note that $1 was then roughly equivalent to ¥97.

[4]The BoJ also announced the introduction of a new purchase operation tool: the purchases of JGBs with yields designated by the BoJ (fixed-rate purchase operations).

sharply increased its holdings of JGBs in 2016, but moderated its pace of bond purchases after 2017. The amounts of five- and 10-year bonds purchased in 2016 are ¥2885.3 billion and ¥2700.8 billion, respectively, and they decrease to ¥1836.7 billion and ¥2647.8 billion in 2018 and to ¥1618.7 billion and ¥1878.1 billion in 2019, respectively, decreases of about 20.0% and 37.4% from 2016. On the other hand, the BoJ's holding ratio continued to increase and reached around 61.9% in 2019, but the growth rate slowed down as the purchased amounts decreased.

Panel B of Table 1 shows the statistics of purchased amounts per security in an operation.[5] The mean purchased amount is about ¥42.0 billion, but the quartile value indicates that the purchased amounts are smaller than this in many operations. The BoJ purchased up to about ¥450.7 billion per bond in an operation, which is about 6.4% of the outstanding amounts of five- and 10-year bonds, respectively, or ¥7 trillion. In the empirical analyses in Section 6.1, we investigate the impact of each purchase operation on the order imbalance in the repo market.

3.2 Central bank lending facility

The BoJ's SLF started to mitigate the tightness of the JGB supply as early as 2004. It functions as follows. Most repo transactions in the repo market were in the T+2 clearing cycle until April 2018, and in the T+1 clearing cycle after May 2018.[6] The BoJ lends bonds from its holdings on the day of settlement (see the timeline in Figure 1). Transactions of the SLF are conducted by multiple-price competitive auctions following the conventional method. In each auction, the BoJ sets an upper limit on the lending rate, taking into account financial market conditions and that the SLF rate is lower than the repo rate in most cases.[7] A dealer who wants to cover a short position but cannot find a counterparty in the repo market can therefore cover it by borrowing the bond from the BoJ on the settlement day, although the cost is mostly higher than in the repo market.

[Figure 1 about here.]

The BoJ changed its lending requirements to ease the deterioration in liquidity caused by its LSAPs. In 2014, lending facility offers were added in the morning and thus become available twice a day. The lending amount per issue was also raised in 2015 and in 2016, and the BoJ set an upper limit on the SLF rate, taking into account financial market conditions.

[5] The BoJ does not disclose its purchased amounts per security in each operation. We estimate these by the BoJ's announcements of amounts held. The BoJ discloses its JGB holdings three times a month. We estimate the amounts purchased in an operation as the increases in holdings. If multiple purchase operations are conducted during the approximately 10-day interval in which the holdings are announced, the increase in the holding amount is divided by the number of operations.

[6] On May 1, 2018, the JGB settlement cycle was shortened from two business days (T+2) to one business day (T+1).

[7] The BoJ states on its website that it sets the upper limit on the SLF rate "to prevent market participants from relying excessively on this facility."

After February 2016, the BoJ clarified the upper limit lending rate to be the uncollateralized overnight call rate rounded off to the first decimal place minus 50 bps.[8] Since the overnight call rate ranged between −10 bps and 0 bps, the upper limits of the lending rates (principled SLF rate) were set to be −50 bps or −60 bps. On June 10, 2019, the BoJ relaxed the conditions for the SLF and reduced the minimum fee rate from 50 bps to 25 bps. It also changed to adopt the Tokyo Repo Rate from the uncollateralized overnight call rate as the prevailing market rate.[9] After this relaxation, the principled SLF rate was set to be about −35 bps. If the repo rate is currently lower than the expected SLF rate, dealers will reasonably choose to forgo borrowing bonds in the repo market and, instead, borrow through the SLF. The central bank lending rates can thus set a lower limit on the repo rate (upper limit on specialness).

[Figure 2 about here.]

Figure 2 shows the principled SLF lending rate, the average SLF lending rate, and the daily amount lent through the SLF. The principled SLF lending rate is calculated as the uncollateralized overnight call rate minus 50 bps for the period before the relaxation and as the Tokyo Repo Rate minus 25 bps after the relaxation. The dotted vertical line denotes the date of the SLF relaxation. The average lending rates through the SLF are the same as the principled lending rates on most trading days. The exceptions are days of high specialness of a specific bond (see also Figure 3). The lending amount through the SLF varies in accordance with demand. Although the average lending amount through the SLF is ¥0.057 trillion, more than ¥0.5 trillion were lent by the BoJ on 1.5% of the days in the sample period.

Due to the relaxed conditions for the SLF, the BoJ's lending rate rose to an average of −35 bps from −50 bps or −60 bps, and the market participants were able to lend at lower cost after the relaxation. However, the lending amount did not increase immediately after the relaxation, one reason being that the repo rate remained high on these days.[10] We investigate the impact of this relaxation on transactions in the repo market in Section 6.2, splitting the sample period into two subperiods by the date of the relaxation of the conditions for the SLF, as follows:

Before relaxation	April 1, 2016 to June 9, 2019
After relaxation	June 10, 2019 to December 31, 2019

We also look at the timing relation between the central bank's purchase and lending operations. As shown in Figure 1, the BoJ's purchase operations are conducted on the same day as bond-specific repo transactions, and the results are announced at 12:00 noon, and more bid orders to cover shorts to purchase operations are thus ordered in the afternoon session of the repo market.[11] If a short seller (borrower) finds a lender in the repo market, the transaction

[8]The uncollateralized overnight call rate released on the previous business day of the SLF lending operation is used.

[9]Figure 3 shows the historical overnight call rate and the Tokyo Repo Rate.

[10]See Figure 3.

[11]Uno and Tobe (2020) show that the number of orders in the afternoon increases on days when the BoJ conducts purchase operations.

is settled the next day (T+1) or the day after (T+2). If a short seller cannot find a lender in the repo market, the short seller will borrow the bond from the BoJ through the SLF held around noon on the settlement day. As already described, the SLF lending rate is decided based on the uncollateralized call rate or the Tokyo Repo Rate on the previous day of the SLF operation, and, thus, after the shortening of the settlement cycle in May 2018, the SLF lending rate can be predicted on the repo transaction day. Even before the shortening, the SLF lending rate can be roughly predicted while trading a repo, because its fluctuations are generally not large, as shown in Figure 2. In other words, borrowers place a repo bid order while considering the expected SLF rate on the settlement day.

3.3 Historical changes in the repo rate

We now look at the historical repo rate during our sample. A repo transaction is categorized as either a general collateral (GC) repo or an SC repo. GC repo transactions cannot specify bonds to be traded mainly for the purpose of raising funds. On the other hand, SC repos are bond-specific transactions for the purpose of lending those bonds, and their rates are priced below the GC repo rates, according to bond availability. The specialness, measured by the difference between the GC and SC rates, indicates information on the state of short selling pressures in the cash market. Many studies, such as Duffie's (1996), show that specialness increases with the amount of short selling activity in the cash market and is driven by the demand for short positions, constraints on the available supply, and the liquidity of the security. The reduction of collateral increases the difficulty of borrowing the bonds. Therefore, the SC rate of a scarce bond should be lower and its degree of specialness should be higher.

[Figure 3 about here.]

Figure 3 shows the time series evolution of the volume-weighted average of the SC rate, the GC rate (Tokyo Repo Rate), and the uncollateralized overnight call rate. The uncollateralized overnight call rate is the reference rate for Japanese yen overnight unsecured transactions in the Japanese market. Although it fluctuates in accordance with the balance of supply and demand, it ranges from -8.1 bps to -0.5 bps during the whole sample period from April 2016 to December 2019. On the other hand, the Tokyo Repo Rate, which is the GC repo benchmark rate, calculated as the average of the GC repo rates reported by reference institutions, is more volatile. GC repos are traded mainly for the purpose of raising funds, and their rates are said to be generally priced at a level close to the call rate, but after the BoJ's introduction of the negative interest rate in January 2016, the GC rate diverged from the call rate in the JGB market. The SC repo rates of a bond fluctuate according to the bond's balance of supply and demand. The SC rates were low (the GC–SC spread is large) in 2017, when the BoJ purchased JGBs aggressively, and the SC rates then gradually rose as the BoJ's purchase amounts decreased. On June 10, 2019, the BoJ switched to link the SLF lending rate to the Tokyo Repo Rate. In the periods before and after this relaxation, the GC rates (as well as SC rates) fluctuated at high levels, and the GC–SC spread was narrow, which is considered to be caused by the reduction of the BoJ's purchases.

4 Model and empirical hypotheses

In this section, we propose a model based on the search-theoretic model of the OTC bond market introduced by Duffie et al. (2002). Our model assumes the central bank to be a lender as well as a purchaser, introducing the role of the SLF to the search-theoretic model. We then calibrate our model and describe the model's predictions and the hypotheses to be examined in our empirical analyses.

4.1 Model calibration

We refer to the models of Duffie et al. (2005) and Ferdinandusse et al. (2020) to clarify the impact of increasing central bank holdings and the cost of the SLF on the repo market.[12] We describe here only an outline of our model. A complete derivation of the model is provided in Section A of the Appendix.

We consider an infinite-horizon steady-state economy. We assume three types of investors in the bond lending market: borrowers, lenders, and a central bank. Borrowers, whose measure is α_{bo}, are short sellers who gain profits by purchasing bonds at low prices and selling them at high prices.[13] They do not hold the bond at the time they decide to sell, and they try to borrow the bond either in the repo market or through the SLF run by the central bank. The lenders, whose measure is α_l, are non–central bank bondholders. They each hold a unit quantity of bonds, making a profit by lending their bonds in the repo market. The ratio $\frac{\alpha_{bo}}{\alpha_l}$ is the ratio of the supply to the demand in the repo market and represents the tightness of the repo market. We now introduce the central bank as a lender, an aspect not covered in previous work. The central bank holds the bonds it purchases to maturity and lends them through the SLF. As already mentioned, since the central bank's SLF rate is set below the SC repo rate in most cases, borrowers borrow bonds through the SLF only when they cannot find a repo counterparty with a better rate than that through the SLF.

We focus only on the bond lending market and consider the utility of the investors obtained through lending or borrowing assets. First, we consider the utility of lenders. We assume lenders need to pay a small holding cost e_l in each period. We also assume that the probability of finding a counterparty depends on the number of such counterparties in the market. A lender finds a borrower with probability $\lambda\alpha_{bo}$ and obtains ω for the bond in a successful repo transaction, where λ is the Poisson arrival intensity. Lenders can switch from patient

[12] Corradin and Maddaloni (2020) extend the model of Vayanos and Weill (2008) by introducing the central bank as the buy-and-hold investor, whereas we model the central bank as a lender of the purchase assets as well. This is a situation that is not considered in their paper. The repo transactions are affected not only by the central bank purchases but lending rate of the SLF. In addition, Corradin and Maddaloni (2020) model both outright and repo transactions. We model only repo transactions for the purpose of clarifying the impact of the central bank purchases and lending on the repo market, but cash market conditions can be reflected in our model parameters. Increasing scarcity in the cash market translates into fewer lenders in the repo market in our model, for example.

[13] Although not all borrowers are short sellers, for the sake of simplicity, we assume no other borrowers in the model.

to impatient agents with probability κ, seeking to sell the asset. A borrower pays a positive lending fee ω to the lender.

We next consider the utility of borrowers (or short sellers), introducing the central bank's SLF to the model. We assume that a borrower tries to borrow a bond (to deliver to the buyer) in the repo market with probability β and to borrow through the SLF with probability $1-\beta$. Whenever there is a sufficient bond supply in the repo market, most lending transactions are carried out in the repo market (β is close to one), because the central bank sets the lending fee above that in the repo market. If the bond is scarce and the rate of the repo market and the SLF rate are at the same level, β will be close to 0.5. We further define a short seller's profit h_s that arises from the sale and purchase of a bond in the bond spot market. In the repo market, the short seller finds a lender with probability $\lambda\alpha_l$ and has a cost ω for the borrowing fee when the repo transaction is successful.

A borrower also has the option to borrow the bond from the central bank. We define the probability of the borrower being successful at borrowing through the central bank's SLF as δ. The term δ is assumed to remain constant for the LSAPs, because a central bank will try to respond to investors' demand for bond lending to maintain the liquidity of the JGB market, which tends to decline due to LSAPs. Let the central bank's lending fee be ω^{cb}, which is set much higher than ω in most cases. When a borrower can borrow the bond in the repo market or through the SLF, the borrower exits the bond lending market and the borrower's (or short seller's) profit is the difference between the profit h_s and the borrowing cost ω (or ω^{cb}). If a short seller cannot borrow a bond to deliver and fails to complete the transaction in the bond spot market with probability q, the short seller must pay the cost of failure $h_f(>0)$, which includes penalties such as a decline in the short seller's credit.

We assume the lending fee is determined through Nash bargaining, which is applied by Duffie et al. (2005) and Ferdinandusse et al. (2020), among others. Under these assumptions, the lending fee ω lies between the utilities of the marginal lender and the marginal borrower in the repo market and is solved by the equations for the utility of the lenders, the utility of the borrowers, and bargaining power.

[Table 2 about here.]

We now calibrate the model using our JGB data set. Table 2 shows the parameter values used in the calibration. We briefly explain how we compute these values. The expected investment horizon of bondholders is matched with the JGB cash market turnover, 1.29. The expected investment horizon of patient bondholders is thus 0.77 years. We set q, the probability of failure, to be 0.80%, which is the amount of failed transactions divided by the amount outstanding.[14] The probability of a successful bid through the SLF, δ, is set high, at 0.999, based on the fact that the amounts of successful bids are below the amounts of competitive bids in only one of 916 SLF operations. The Poisson intensity of the search process, λ, is a constant in this model, and we set it to 100,000, which means that, if the

[14]The amount of failed transactions in the JGB market is ¥7.0 trillion per year and the amount outstanding of nominal JGBs is ¥880 trillion, averaged over our sample period.

measure of lenders is one, it takes 1/400th of a business day, on average, to find a lender. We set the profit arising from the sale and purchase of a bond, h_s, to 0.036%, which is matched to the average yield decline per year of the 10-year on-the-run bond. The cost of failure, h_f, which includes penalties such as a decline in the short seller's credit, is set to 0.03. The lender's search cost e_l is set to the low value of 0.00001. On June 10, 2019, the BoJ relaxed the conditions for the SLF and reduced the minimum fee rate from 50 bps to 25 bps, and the SLF rate rose from −50 bps or −60 bps to around −35 bps. We thus set the central bank's lending fee, ω^{cb}, to 0.5% before the relaxation period and to 0.35% after the relaxation. The risk-free rate r is set at −0.035% by averaging the overnight call rate throughout the sample period. The bargaining power of borrowers, ϕ, is set to 0.5.

The SLF operated by the central bank provides alternatives for short sellers covering their short positions. In our model, β, a borrower's probability of choosing to borrow a bond in the repo market, is a key parameter that indicates the repo market situation. When the bond supply in the repo market is sufficient and the borrowing fee will be more favorable than that of the SLF, β is close to one. On the other hand, if bond scarcity increases and the rate of the repo market and the SLF rate are at the same level, borrowing in the repo market and through the SLF will be equivalent for borrowers, so β will be close to 0.5. We assume β follows the following error function:

$$\beta = \left(\frac{1}{2} \left(1 - Erf[\eta_1(\omega - \eta_2)]\right) \right)$$

where $Erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)dt$, and we set the parameter η_1 to 550 before the relaxation period and to 750 after the relaxation, which is matched to the observed β.[15] We also set η_2 equal to ω^{cb}.

The measure of lenders, α_l, is an indicator of bond scarcity in the repo market. Prolonged LSAPs decrease the amount of bonds held by private investors and thus decreases the number of lenders in the repo market. The other factor that tightens the supply–demand balance is a rapid increase in bond demand, such as the demand on the end day of a quarter or the demand for the central bank's purchase operation. The ratio $\frac{\alpha_{bo}}{\alpha_l}$ represents the tightness of the repo market, and it must change on a daily basis or within a day. We calibrate the model for different α_l values while fixing α_{bo} to $\frac{1}{2}$ to investigate the impact of the tightness on the lending fee. In our search-theoretic model, a borrower meets a lender with Poisson intensity $\lambda\alpha_l$, which means a borrower needs $1/\lambda\alpha_l$ years to finds a lender, on average.[16] In the calibration, we set α_l to 0.1, 0.05, and 0.01, where 12, 24, and 120 minutes, respectively, are needed for a borrower to find a lender.[17] We run the model for the two periods, before

[15] About 38.1% of bonds with a 0–5 bps rate difference between the SC rate and the SLF rate are lent through the SLF, based on our data set. The proportion decreases to about 5% as the rate difference increases to about 20 bps.

[16] According to our model, the repo lending fee increases as the measure of lenders, α_l, decreases. If we set λ to 100,000, as listed in Table 2, and the measure of lenders α_l is one, a borrower needs 1/100,000 years (1/400th of a trading day) to find a lender. If the measure of lenders α_l becomes 0.1, a borrower needs 1/10,000 years (1/40th of a trading day, or about 12 minutes) to find a lender in the repo market.

[17] In the empirical analyses in Section 6.3.1, we calculate the average duration time using intraday data

relaxation and after relaxation, to investigate the impact of the SLF fee relaxation. We further run the model for the case in which the central bank does not have a lending facility.

[Table 3 about here.]

Table 3 shows the simulated lending fee, ω, and the corresponding probability of a borrower borrowing a bond in the repo market, β. Column (1) of Table 3 shows the cases in which the central bank does not have a lending facility. The calibrated lending fees for α_l values of 0.1, 0.05, and 0.01 are 9.07 bps, 17.70 bps, and 72.53 bps, respectively. Increasing bond scarcity and the tightness of the repo market raise the lending fee. Columns (2) and (3) of Table 3 show the cases in which the central bank has an SLF. The calibrated lending fees for α_l values of 0.1, 0.05, and 0.01 are 9.19 bps, 20.06 bps, and 52.52 bps, respectively, before relaxation. The SLF works to curb increases in the lending fee ω. When there are few lenders in the repo market (see the rows where $\alpha_l = 0.01$), the rate that would rise to 72.53 bps without an SLF is constrained at 52.52 bps. After relaxation, the calibrated lending fee for α_l values of 0.1, 0.05, and 0.01 are 9.42 bps, 24.72 bps, and 37.85 bps, respectively, and are suppressed at much lower fees.

The relaxation of the SLF fee affects borrowers' selection probability, as well as repo lending fee. The calibrated probability of borrowing in the repo market, β, for an α_l value of 0.1 is above 99%, but it declines to 42.24% before the relaxation period and to 38.11% after the relaxation as the measure of lenders decreases. A decline in lenders and an increase in bond scarcity increase the utilization of SLFs. Based on these calibration results, we next develop our hypotheses.

4.2 Hypotheses

In this section, we present the research questions to be tested in the empirical analyses.

After the introduction of QQE, the BoJ purchased large amounts of government bonds, holding them on its balance sheet, as shown in Table 1. Under the QQE program, the central bank repeated the purchase operation in the form of an auction. The demand for repo transactions was increased to cover short positions created in response to the central bank's purchase operations. D'Amico et al. (2018) quantify the scarcity value of Treasury collateral and Corradin and Maddaloni (2020) show theoretically that the demand for repo transactions is high when the central bank purchases government bonds. To investigate more directly the effect of the BoJ's purchase operations on repo tightness, we focus on the ratio of bid and offer orders in the repo market. According to our search-theoretic model, an increase in the number of borrowers will tighten the demand–supply situation, that is, $\frac{\alpha_{bo}}{\alpha_l}$ increases as α_l decreases, making counterparties harder to find. We thus propose the following hypothesis.

Hypothesis 1 *Central bank purchase operations increase repo bid orders. The greater the scarcity, the larger the demand for repo transactions.*

on repo transactions. The duration time for bid orders to cover shorts is 5.17–14.92 minutes, on average, according to bond scarcity (see Table 8).

The SLF operated by the central bank can mitigate a tight demand–supply situation. It provides alternatives for short sellers covering their short positions. However, the lending rate through the SLF is set above the normal rate obtainable in the repo market, so that it works as a last resort to avoid short seller failure.

After February 2016, the central bank's SLF lending rate was at the level of −50 bps—or −60 bps—and the rate was changed to −35 bps in June 2019. This change affected traders' choice of transaction places and anticipation of the maximum lending fee. For example, if an SC rate was around −35 bps before June 2019, a borrower would trade in the repo market; that is, β, a borrower's probability of choosing to borrow in the repo market in our model would be high. After the relaxation, the lending cost of the repo market and that of the SLF become comparable, so that β decreases. The relaxation is expected to increase the volume of lending through the SLF. As for the SC rate, the calibration results in Table 3 show that the repo lending fee ω increases as β decreases, to a level around the SLF lending fee, ω^{cb}. The SLF lending rate creates a ceiling effect on the SC rate. We account for these effects by considering the SLF explicitly, which is a novel contribution of our paper. The BoJ's relaxation of the SLF conditions provides an opportunity to estimate the relation between LSAP and SLF at different settings.

Hypothesis 2 *The difference between the SC rate and the SLF rate affects dealers' choice between the repo market and the SLF. The higher the central bank's SLF rate, the greater the likelihood of borrowing from the SLF.*

Hypothesis 3 *The central bank's SLF has a ceiling effect on the SC repo rate. The relaxation of SLF conditions induces a low SC rate.*

Empirical works such as those of D'Amico et al. (2018) and Kinugasa and Nagano (2017) investigate the relation between the daily SC rate (or specialness) and LSAPs to quantify the scarcity of government bonds. We quantify frictions with more concreate measures that have not been tested empirically. Our order submission data enable us to keep track of the rate changes of bid and offer orders. We measure the time necessary to complete transactions and the rate concession amounts of lenders and borrowers separately. Longer duration times indicate greater difficulties of borrowing in the repo market, as do larger rate concession amounts for bid orders. As in Vayanos and Weill (2008), repo liquidity can be measured by duration times in the search theoretic model. We test their model prediction and show search frictions in the repo market.

We further test the bargaining power of borrowers (or lenders) in our search-theoretic model. We assume that the rate concessions of borrowers and lenders each represent bargaining power, and we consider the ratio of the rate concession amounts for offer orders to the aggregate rate concession amounts as our proxy for the bargaining power of borrowers. Larger relative concession amounts indicate weaker bargaining power. We expect the central bank's relaxation of SLF conditions to weaken lenders' bargaining power.

Hypothesis 4 *The greater the scarcity, the longer (shorter) a bid (offer) order takes to be filled. The bid rate for a scarcer bond is subject to greater concession during trading hours.*

Hypothesis 5 *Lenders' bargaining power increases with the scarcity caused by the central bank's LSAPs and weakens with higher SLF rates.*

5 Market data and variables for empirical analysis

5.1 Market data

Our data for repo transaction are obtained from the electronic platform provided by JBOND. The order submission data set, which includes the date, time, bond code, order number, rate, and volume, allows us to keep track of the initial and filled rates for individual bid/offer orders. For the empirical analyses, we use overnight SC repo transactions, which account for 99.0% of all the transactions in our sample. Our universe of empirical analysis data covers five- and 10-year JGBs. We select these because investors in the government bond market are segmented and the outstanding amounts and BoJ holdings of these bonds are large.[18]

The electronic platform data we rely on differ from the model's assumption of an OTC market. The data from the platform, however, mirror the search process of an OTC market. Traders typically submit a bid order with a favorable price while they search for the same bond in the OTC market and negotiate a price at which they can borrow. Activities in the OTC market are not observable, but the quoting behavior and execution status on the electronic platform reflect the difficulty of search activity in the OTC market. If a trader finds a counterparty for the shorting bond, the trader immediately cancels the order submitted to the platform, to avoid a double execution. The trader can change bid quotes to expedite the order's completion. Duration time and concessions observed on the platform are unique empirical evidence of the cover risk.

5.2 Variables

We now present our measures to test the hypotheses. In the search-theoretic model applied by Duffie et al. (2002) and many others, including us, the probability of a particular borrower finding a lender is defined as a product of the Poisson intensity and the number of lenders. The increasing scarcity of the JGB market due to the BoJ's LSAPs diminishes the number of

[18]The distributions of investors for short-term, mid-term, long-term, and super-long-term bonds are very different. Foreign investors account for about 27% of outright transactions for mid-term (two- or five-year maturity) and long-term (10-year maturity) JGBs, and insurance company and pension funds account for 16% of them. As for super-long-term (20-, 30-, and 40-year maturity) bonds, foreign investors account for about 13% of outright transactions, and insurance company and pension funds account for 27% of them. As for short-term Treasury discount bills, foreign investors account for about 44% of outright transactions, and insurance company and pension funds account for 4% of them, based on the transactions in April 2018.

lenders and each purchase operation increases the number of borrowers in the repo market. We define the order imbalance $imb_{n,t}$ as the proportion of bid orders among all orders of a bond n on day t.[19] Increasing scarcity and a high demand for a repo transaction are both expected to raise the bid ratio.

To investigate the impact on the repo rate, we use the intraday SC rate as well as the daily SC rate. The daily SC rate of security n on day t, $sc_{n,t}$, is calculated by the volume-weighted SC rate and used for regression analysis, whereas the intraday rate is used for aggregating orders. We further define $rdiff_{n,t}$ as the difference between the daily SC rate and the SLF lending rate, to investigate the competitive relation between the repo market and the SLF. A smaller $rdiff_{n,t}$ means higher competition, and if $rdiff$ takes a negative value, SLF lending is a better choice than a repo transaction for the bond short seller.

We propose two measures for examining search friction in the repo market. We measure delays in order execution and the incremental execution costs for all orders. We track each order's execution, cancellation, or modification by the order ID and measure the time interval and rate change between a new order placement and its execution. As far as we know, we are the first to investigate search friction by tracking the search costs of individual repo orders. We define the duration time as the time interval between an order entry and its execution. We also define the rate concession amount $rconcession_{n,t}$ as the associated difference between the initial and final rates. We examine these two measures for bid and offer orders separately. Bid orders include cases in which bidders must cover their short positions, whereas offer orders are aimed at raising money using bonds in hand.

In our search-theoretic model, the lending fee is determined by Nash bargaining. We attempt to estimate the bargaining power of borrowers and lenders. We consider that, if lenders (borrowers) have greater bargaining power, they can offer lower concessions and we assume that the relative rate concession amounts of borrowers (lenders) represent the bargaining power of lenders (borrowers). Our proxy for the bargaining power of borrowers is defined as the ratio of the absolute value of the rate concession amounts for offer orders to the aggregate rate concession amounts. We calculate this ratio for each security for each day.

As seen in Table 1, the BoJ's LSAPs have a significant impact on the bond supply. We construct our scarcity variable as the percentage of the amount outstanding of the BoJ's security holdings n on day t, $h_{n,t}$. This variable indicates whether sufficient bonds exist in the market. To control for the outstanding amount of a bond, we also define $outstanding_{n,t}$ as the logarithm of the outstanding amount of security n on day t. We also define $po_{n,t}$ as the amount of the targeted security n purchased in the operation on day t, to examine the effect of being purchased in each purchase operation.[20] After years of LSAPs, market liquidity conditions have decreased further and further, such that the demand for procuring bonds in the repo market and repo specialness have increased.

[19] Corradin and Maddaloni (2020) defines the variable repo imbalance as the difference between aggregate reverse repo and financing repo transactions (as a percentage of the outstanding amount) based on the information on the trading direction. Our order imbalance measure $imb_{n,t}$ is calculated on an order basis, not a transaction basis. It should show imbalance between borrowers ' and lenders ' demand more clearly.

[20] The variable $po_{n,t}$ represents estimated amounts rather than exact amounts. See footnote 5.

The total traded amount of a bond is defined as $tradeda_{n,t}$, the cumulative amount of bond n traded on day t in the repo market. A larger traded amount suggests that more borrowers and lenders meet in the repo market that day. Repo specialness is sensitive to the JGB auction cycle, as indicated in the literature on US Treasury bonds (e.g., Sundaresan (1994), Keane (1995), D'Amico et al. (2018)). To control for this effect, we use an on-the-run dummy, *ontherun*, for the most recently issued bond; an ex-on-the-run dummy, *exontherun*, for the second most recently issued bond; and *age*, which is defined as the number of years since issuance or the most recent reopening. We also construct a dummy variable for the cheapest-to-deliver bond, *ctd*, to control for its active trades. As Figure 3 shows, repo rates have many spikes, and many of these are on days of high cash demand, such as at the end of quarters and months and in the middle of months. We add date dummies to our panel regression model in Section 6 to control for these effects, as well as other date factors, such as program announcements.

6 Empirical results

6.1 Impact of the central bank's purchase operations

In this section, we consider the effect of the central bank's LSAPs on repo transactions. We investigate the impact of bond purchases in each operation as well as bond scarcity on repo orders.

Hypothesis 1 *Central bank purchase operations increase repo bid orders. The greater the scarcity, the larger the demand for repo transactions.*

6.1.1 Impact of purchase operations on order imbalance

First, we consider the effect of the central bank's LSAPs. Each central bank purchase operation implemented in the form of an auction influences dealer activity in the repo market. The BoJ announces auctions to dealers at 10:10 a.m. and accepts their bids until 11:40 a.m., and the bidders are notified of the results around 12:00 p.m.[21] If an auction bidder does not hold the bond to be sold to the central bank, the bidder most likely locates the bond in the repo market. Thus, central bank purchase operations must increase the demand for repo transaction. On the other hand, long-term implementation of LSAPs increases the scarcity of bonds. LSAPs decrease the number of lenders (α_l) and the bidder is more likely to locate the bond in the repo market when the particular bond is already scarce, and thus the tightness of the repo market ($\frac{\alpha_{bo}}{\alpha_l}$) increases. We expect an increase in the number of bid orders relative to offer orders.

[Table 4 about here.]

[21]See also Figure 1.

First is to examine the cumulative effect of the LSAPs on order imbalance. Table 4 shows the relation between the holding ratio of the BoJ and order imbalance. Our order imbalance is based on the number of new bid or offer orders. We do not include the number of revisions and cancellations. The interaction between order imbalance and the holding rate is clear. When the holding ratio is below 20%, the average order imbalance is 45.6%, which means the number of orders submitted by lenders is larger than that submitted by borrowers. However, when the holding ratio increases to between 0.6 and 0.8, the imbalance is 53.2%, which means the number of orders submitted by borrowers is larger than that submitted by lenders, and the imbalance increases to 57.9% when the BoJ holds over 80% of the bond outstanding. The higher the holding ratio, the larger the imbalance. The average order imbalance levels observed in the five holding ratio ranges are significantly different according to the Welch two-sample t-test, so the holding ratio affects the level of order imbalance. The high BoJ holding rate means less float in the cash market, so that borrowers ought to search for a bond in the repo market. A shortage of lenders increases order imbalance, as expected.

Bond scarcity and order imbalance have a strong relation. Therefore, we cannot accurately analyze the impact of each purchase operation on the repo market unless we consider this relation. Controlling for this relation, we now investigate the impact of each purchase operation by running a regression.

Each purchase operation by the central bank increases the number of borrowers, because an auction bidder locates the bond in the repo market if the bidder of a purchase operation does not hold the bond to be sold. We thus expect an increase in the number of bid orders relative to offer orders for purchased bonds. We investigate the impact of each purchase operation by running the following regression:

$$imb_{n,t} = \beta_1 h_{n,t} + \beta_2 h_{n,t}^2 + \beta_3 logpo_{n,t} + \beta_3 logpo_{n,t} h_{n,t} + \psi_n + \zeta_t + \epsilon_{n,t} \quad (1)$$

where $imb_{n,t}$ is the order imbalance of security n on day t. The regressors are the BoJ's holding rate $h_{n,t}$ and its squared value, the logarithm of the amount of security n purchased by the central bank on date t, $logpo_{n,t}$, and its interaction term with $h_{n,t}$. Our model includes security-level fixed effects ψ_n and time dummies ζ_t to control for security- and date-specific effects, respectively. The term $\epsilon_{n,t}$ is the error term. We also include control variables such as the number of years since the issue or reopening date, $age_{n,t}$; the on-the-run bond dummy, $ontherun_{n,t}$; the ex-on-the-run bond dummy, $exontherun_{n,t}$; the cheapest-to-deliver bond dummy, $ctd_{n,t}$; and the logarithm of the outstanding amount of security n on day t, $outstanding_{n,t}$, for model (M2).

[Table 5 about here.]

[Figure 4 about here.]

Table 5 reports the impact of the central bank's purchase operations on the order imbalance. Column (M1) of Table 5 presents the results for the model described in Eq. (1). The estimated coefficient of the purchased amount is 0.0051, and that of the interaction term between the purchased amount and the holding ratio is significantly negative. The larger the BoJ purchase, the higher the order imbalance. A purchased bond with greater scarcity causes higher order imbalance. The results support Hypothesis 1. The results indicate that each central bank purchase operation affects the supply–demand balance in the repo market through short selling activity. Many of the bonds that the BoJ purchases in large amounts are considered to be less scarce bonds, such as on-the-run bonds. Column (M2) in Table 5 shows the results of the regression, controlling for these effects. We obtain similar estimates for the coefficients of $h_{n,t}$, $h^2_{n,t}$, $logpo_{n,t}$, and its interaction term with $h_{n,t}$.

We also calculate the contribution of the purchase operation variables on order imbalance based on model (M1), which has a higher R-squared value. Figure 4 shows the calculated contribution.[22] The calculated contribution shows that order imbalance grows as the purchased amounts in an operation increase. However, the increase in the order imbalance is more pronounced for less scarce bonds. If the BoJ purchases ¥100 billion of a bond in an operation, the imbalance rises by 3.54%, 2.59%, 1.59%, and 0.68% for bonds with holding rates of 0%, 20%, 40%, and 60%, respectively. The impact of the BoJ's bond purchase of ¥100 billion is larger for less scarce bonds, which is opposite our expectation. Possible reasons for this are that bonds with few holdings by traders other than the BoJ cannot be sold in large amounts in a purchase operation, so that the impact of purchases is smaller for these bonds.

6.1.2 Threshold analysis of purchased amounts

The results in Section 6.1.1 indicate that the ratios of bid orders are explained by the purchased amounts, as well as bond scarcity. In this section, we introduce a threshold dummy variable that depends on the amounts purchased by the BoJ, to investigate how much of the bonds the BoJ purchases in an operation when bid orders increase significantly.

We set the amount purchased by the BoJ, $po_{n,t}$, as our threshold variable and define the dummy variable

$$dpo_{n,t}(\chi) = I\{po_{n,t} > \chi\} \tag{2}$$

where $I\{\cdot\}$ is the indicator function.

Excluding the term of the logarithm of the BoJ's purchased amount, $logpo_{n,t}$, and its interaction term with the holding rate $logpo_{n,t}h_{n,t}$ from Eq. (1) and introducing the threshold dummy variable $dpo_{n,t}(\chi)$,[23] we obtain

$$imb_{n,t} = \beta_1 h_{n,t} + \beta_2 h^2_{n,t} + \beta_3 dpo_{n,t}(\chi) h_{n,t} + \psi_n + \zeta_t + \epsilon_{n,t} \tag{3}$$

The coefficient of $dpo_{n,t}(\chi)h_{n,t}$ represents the threshold effect, where large amounts of BoJ purchases have an impact on order imbalance. We examine the significance of β_3 in Eq. (3) to test the hypothesis regarding the sensitivity of the imbalance with a specific threshold value of BoJ purchases. We test the significance of the threshold χ at intervals of ¥1 billion.

[22]The contribution is calculated by $\hat{\beta}_1 h_{n,t} + \hat{\beta}_2 h^2_{n,t} + \hat{\gamma}_1 logpo_{n,t} + \hat{\gamma}_2 logpo_{n,t} h_{n,t}$, where $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are the coefficient estimates of (M1).

[23]See Greene (1999) p. 342

[Figure 5 about here.]

Figure 5 shows the results of the threshold dummy variable analysis and the sequential p-values of the significance tests. All the coefficients β_3 are estimated to be positive. The t-test shows that the coefficients are significantly different between the purchase amounts of ¥9 billion and ¥35 billion. The ratio of bid orders of a bond increases significantly when the BoJ purchases ¥9 billion of the bond or more in a single operation, which indicates a change in sensitivity when the amounts purchased by the BoJ exceed ¥9 billion (equivalent to 1.4% of the outstanding). Hypothesis 2 is verified, and, when bonds are purchased by the BoJ, the proportion of bid orders rises in the repo market. The results suggest that the central bank should pay greater attention when a purchase operation exceeds ¥9 billion, which could have an unusually large impact on the repo rate.

6.2 Role of the SLF

The BoJ, which holds about half of the government bonds, has strengthened the SLF to alleviate the deterioration of liquidity. This section examines the functions of the SLF and its effect on the repo market.

Hypothesis 2 *The difference between the SC rate and the SLF rate affects dealers' choice between the repo market and the SLF. The higher the central bank's SLF rate, the greater the likelihood of borrowing from the SLF.*

Hypothesis 3 *The central bank's SLF has a ceiling effect on the SC repo rate. The relaxation of SLF conditions induces a low SC rate.*

6.2.1 Impact of the relaxation of the SLF conditions on repo transactions

We first examine the interaction between the rates in the central bank's SLF and the repo market. As noted in Section 3.2, the BoJ facilitates the lending of bonds in a manner that complements repo transactions. Since February 2016, the rate set by the BoJ is linked to the uncollateralized overnight call rate, so that the actual lending rates fluctuate around -50 bps to -60 bps. If the SC rate in the repo market is lower than -50 bps (or -60 bps), borrowing from the BoJ becomes a reasonable choice for dealers. Therefore, the SC rate has a ceiling imposed by the SLF. On June 10, 2019, the BoJ reduced the minimum fee and set the SLF rate to be about -35 bps. If the central bank's lending facility set the ceiling of the SC repo rate, the ceiling must change from around -50 bps to around -35 bps due to this fee relaxation. We test whether this change in the SC rate is observed along with the relaxation. As noted in Section 3.2, a short seller places a bid order in the repo market and tries to find a counterparty willing to trade at a better rate than that from the SLF. Short sellers' behaviors to cover short positions must depend on the rate difference *rdiff* between the repo market and the SLF. We thus calculate the number of orders by the difference in the *rdiff* category, as well as the SC rate category.

[Table 6 about here.]

Panel A of Table 6 shows the average number of bid orders per day for each SC rate level and their proportions. We focus on bid orders in the aim of revealing the impact of the BoJ's LSAPs on short-selling activities in the repo market. Before the relaxation of the SLF conditions, the proportion of SC rates above −20 bps was 73.0%, but it rose to 95.4% after the relaxation. The increasing proportion after relaxation reflects two changes: one is the weaker LSAPs conducted by the BoJ, and the other is the effect of the relaxation. Before the relaxation, when the SLF lending rate was −50 bps or −60 bps, 4.4% of the repo orders were placed below −50 bps; but after the relaxation, when the SLF rate changed to −35 bps, almost no orders are less than −50 bps. This indicates the ceiling effect on the repo rate.[24]

To reveal the mechanism of relaxation, we calculate the number of orders by the difference between the SC rate and the SLF lending rate, *rdiff* (see Panel B of Table 6). Before the relaxation, 88.2% of the trades were above the 25-bp rate difference, but, after the relaxation, this result reduced to 51.2% and the remaining 48.8% traded at a rate difference of less than 25 bps. Transactions now take place within a much tighter range. Hypothesis 3 is supported. Market participants place repo orders with a strong awareness of the SLF rate.

We find a ceiling effect for the periods both before and after the relaxation. The ceiling level changes according to the SLF lending rate, which indicates market participants modify their rate expectations, taking care not to exceed the SLF rate. The SLF lending rate set by the central bank has a great influence on borrower behavior.

6.2.2 Choice between the SLF and repo transactions

We next investigate the characteristics of bonds lent through the SLF. Borrowers who cannot borrow a bond in the repo market will submit a bid for the lending operation through the SLF on the settlement day. We investigate the determinants of the probability of lending through the SLF. We establish a probit model to predict whether a bond is lent through the SLF conditional on the SC rate level. We calculate the proportion of bonds lent through the SLF to investigate the impact of the SLF relaxation. Does a higher SLF rate lead to greater lending from the BoJ? To shed light on the determinants of SLF lending, we further establish a model to predict the SLF lending probability conditional on several indicators, such as the BoJ holding ratio as a proxy for bond scarcity, order imbalance, and amounts purchased in an operation. In the estimation, we again use bid orders for five- and 10- year JGBs. The model is

$$P(y_{n,t} = 1) \quad = \quad \Phi(\beta_0 + \gamma^j Z^j_{n,t} + \epsilon_{n,t}) \tag{4}$$

where $y_{n,t}$ is a binary variable that equals one if the bond is lent through the SLF on the settlement date, and zero otherwise, and $\Phi(\cdot)$ is the cumulative normal probability density function. The term $Z^j_{n,t}$ includes the SC rate of security n on day t, with $sc_{n,t}$ for model (1)

[24]There are only three orders less than −50 bps in the seven months after the relaxation.

or (2); the difference between the SC rate and the SLF rate, with $\mathit{rdiff}_{n,t}$ for model (3) or (4); and several variables that determine the repo rate for model (5), such as the BoJ's holding rate, $h_{n,t}$; and the order imbalance, $imb_{n,t}$.[25] Our model includes time dummies ζ_t to control for date-specific effects and security fixed effects ψ_n for models (2), (4), and (5). The term $\epsilon_{n,t}$ is the error term. We run the regression on a daily basis—and not a deal basis—to prevent the effects of specific bonds with high numbers of transactions from becoming too strong, except for the deal basis analysis. The lower the SC rate, the higher the SLF lending probability is expected to be, and the closer the SC rate is to the SLF rate (the smaller the difference *rdiff*), the higher the SLF lending probability is expected to be.

[Table 7 about here.]

Table 7 reports the maximum likelihood estimation results of the panel probit model. Panel A shows the results for the period before the relaxation, and Panel B shows those for the period after the relaxation. Each upper panel presents the coefficient estimates and each lower panel presents the marginal effects. We calculate the marginal effects averaged from the model estimates for each of the SC rate ranges for model (1) or (3), and those averaged from the model estimates for each of the *diff* ranges for model (2) or (4). The marginal effects of each explanatory variable for model (5) are calculated from the model estimates at the sample mean.

Consistent with our expectations, the coefficients of the SC rate are negative and significant for both periods. The marginal effect of model (1) for the period before relaxation (column (A1)) shows the probability of the SLF lending rising by 0.13% (0.18%) as the SC rate declines by 1 bp for an SC rate above −10 bps (for an SC rate between −20 bps and −10 bps). Then it rises to 0.93% (1.25%) as the SC rate declines by 1 bp for an SC rate between −50 bps and −40 bps (for an SC rate below −50 bps). The lower the SC rate, the higher the SLF lending probability. When we compare the marginal effects before and after relaxation (columns (A1) and (B1)), the rise in the marginal effect becomes sharper after relaxation. The probability of SLF lending rises by 0.37% and 0.62% before relaxation and by 0.70% and 1.15% after relaxation as the SC rate declines by 1 bp for bonds with $-30 < sc \leq -20$ and $-40 < sc \leq -30$, respectively. The probabilities of SLF lending are expected to double after relaxation at these SC rate ranges, which indicates dealers' choice between the repo market and the SLF changing due to the relaxation of the SLF conditions. Hypothesis 2 is validated.

We next look at the interaction between the SLF lending probability and the rate difference between the repo rate and the SLF rate for models (3) and (4). The marginal effects of the rate difference show that the likelihood of SLF lending rises by 1.00% before relaxation and by 1.03% after the relaxation, respectively, as the rate difference reduces by 1 bp for bonds with *rdiff* below 10 bps (columns (A3) and (B3)). The SLF lending probability estimates have similar values for both periods when the repo rate is within 10 bps of the SLF lending rate, which suggests that the most important determinant of the SLF lending probability is

[25]We calculate the correlations between all the regressors of model (5). The correlation between $h_{n,t}$ and $imb_{n,t}$ is 0.134, and the other correlations are not too high.

the rate difference between the repo market and the SLF. The increase in the SLF lending probability for a 1-bp narrowing of the rate difference becomes smaller as the rate difference widens to above 10 bps or to above 20 bps. The smaller *rdiff* is, the higher the SLF lending probability, as expected. The coefficient estimates for two-way fixed effects regression for the SC rate model in columns (A2) and (B2) and those for the rate difference models (A4) and (B4) have the same significance and sign as in the pooled regression.

We further investigate the determinants of the likelihood of SLF lending, using a variety of variables, such as the BoJ's holding ratio, order imbalance, and the on-the-run dummy (see columns (A5) and (B5)). The coefficients of the BoJ's holding ratio are positive and significant before relaxation. The average marginal effect shows the probability of SLF lending rising by 2.394% before the relaxation as the holding rate increases by 10%. Bonds that are scarcer due to the central bank's purchases are more likely to be lent through the SLF. Order imbalance has a positive impact on the determination of SLF lending before the relaxation. The probability of SLF lending increases by 0.117% with a 10% higher bid proportion. In the period after relaxation, however, the BoJ's holding rate and order imbalance are not significant, but the amounts purchased have a positive impact on the SLF lending probability. Larger amounts purchased by the BoJ lead to a higher SLF lending probability. On-the-run bonds that have a high demand for repo transactions tend to be lent more through the SLF before and after the relaxation. The marginal effects show that on-the-run bonds have an 8.74% higher SLF probability than other bonds before the relaxation, and a 4.38% higher probability after the relaxation. Although the number of bonds lent through the SLF accounts for only a small portion of the bonds ordered in the repo market, various factors, such as bond scarcity and supply and demand in the repo market, affect SLF lending.

6.3 Friction measures and the difficulty of execution

Studies using daily data, such as those of D'Amico et al. (2018) or Musto et al. (2018), have shown that scarcity due to the central bank's LSAPs reduces the SC rate or specialness, suggesting that scarcity increases search friction, resulting in a lower SC rate. In this section, we propose friction measures that are more directly related to the nature of search frictions using deal-based transaction data. We also test the bargaining power of borrowers (or lenders) in our search-theoretic model.

Hypothesis 4 *The greater the scarcity, the longer (shorter) a bid (offer) order takes to be filled. The bid rate for a scarcer bond is subject to greater concession during trading hours.*

Hypothesis 5 *Lenders' bargaining power increases with the scarcity caused by the central bank's LSAPs and weakens with higher SLF rates.*

6.3.1 Order duration and LSAPs

Given greater scarcity, a dealer with a short position is expected to be forced to spend longer locating a specific bond. In this analysis, we focus on bid orders in the aim of revealing the impact of the BoJ's LSAPs on short selling activities in the repo market. We compute the average duration of bid orders by splitting the sample based on bond scarcity and investigate the effects of LSAPs on the search process.

[Table 8 about here.]

Table 8 shows the relation between the holding ratio of the BoJ and the duration of order execution. When the holding ratio is below 20%, the average duration for bid orders is 6.23 minutes before the relaxation and 6.05 minutes after the relaxation. The duration is the shortest for bonds with a holding rate between 20% and 40%. The duration increases sharply to 11.53 minutes before the relaxation and 14.92 after the relaxation when the holding rate rises above 80%. The higher the holding ratio, the longer the duration of order execution, which supports Hypothesis 4. LSAPs increase dealers' search friction. In the model of Vayanos and Weill (2008), borrowers' longer duration times are associated with lower competition between lenders and greater lending fee in the repo market. We confirm this relation using actual data. Prolonged LSAPs decrease the amount of bonds held by private investors and thus decreases the number of lenders in the repo market, which cause less competition between lenders. Borrowers need longer time to find a lender.

The changes in duration between before and after the relaxation show no significant difference when the holding rate is 40% or less, but, when the holding rate is above 40%, durations are longer after the relaxation of the SLF conditions. One possible reason is that the SLF rate relaxation induced by the BoJ leads to an increase in the number of dealers who are spending more time searching without rushing to fill their orders.

6.3.2 Bargaining power and bond scarcity

We next consider another indicator of search friction: rate concession amounts. Given greater scarcity, a dealer with a short position will be forced to pay higher prices than the initial price to avoid failure to deliver. In our search-theoretic model, if a repo transaction occurs, the lending fee is set so that the lender receives a fraction $\phi \in [0,\ 1]$ and the borrower receives $1-\phi$ of the total surplus. If borrowers' and lenders' bargaining power are even ($\phi = 0.5$), they will make mutual concessions. If the borrowers' bargaining power weakens ($\phi < 0.5$), they will have to make greater concessions. We consider the relative rate concession amounts for bid (offer) orders as indicators of the relative bargaining power of the lenders (borrowers). The borrowers' bargaining power ϕ is then estimated by the ratio of the rate concession amount for offer orders to the aggregate rate concession amount for bid and offer orders.

Table 9 shows the average rate concessions for bid orders and offer orders separately. The average rate concession amounts are greater for scarcer bonds. The concession amounts for bid orders are 0.117–0.143 bps for bonds that comprise less than 60% of the BoJ's holdings before the relaxation, increasing to 0.351 for those comprising 60–80% of the BoJ's holdings and to 0.401 bps for those comprising more than 80%. We obtain similar relation after the relaxation. Bond borrowers need to concede more for scarcer bonds, which supports Hypothesis 4.

[Table 9 about here.]

Our proxy for the bargaining power of borrowers is shown in the bargaining power column in Table 9. We expect the calculated proxy value to be lower for scarcer bonds, because a lender has more bargaining power in those bonds. In the period before the relaxation, the bargaining power is estimated to be 0.530 for bonds with $0 \leq h < 0.2$. Then the power declines to 0.438 for bonds with $0.4 \leq h < 0.6$ and to 0.447 for those with $0.6 \leq h < 0.8$, and the t-test exhibits significantly stronger power for lenders. This result indicates the aggressive stance of the offer side and that the bargaining power of lenders is stronger than that of borrowers, reflecting the increasing scarcity of government bonds, though the bargaining power for bonds with $0.8 \leq h$ exhibits no significant difference. These results support Hypothesis 5. On the other hand, after the relaxation, the bargaining power estimates increase, which indicates the bargaining power of borrowers (or short sellers) becomes stronger. The relaxation of the SLF conditions changes the relative bargaining power of borrowers and lenders, and lenders need to concede greater rates for execution after the relaxation. The lower rate difference between the repo rate and the SLF rate leads to the longer duration and stronger bargaining power of the borrowers. The central bank's SLF affects borrowers' and lenders' behavior during trading hours.

6.3.3 Ceiling effect on the rate concession

Using rate concession amounts, we reconfirm the ceiling effect of the central bank's SLF. We test whether the rate concession is smaller when the ordered SC rate is closer to the SLF lending rate. We include four dummies of different *rdiff* levels and run the following regression:

$$\begin{aligned} rconcession_{n,t} &= \beta_0 + \beta_1 h_{n,t} + \beta_2 tradeda_{n,t} + \sum_j \gamma_1^j Ages_{n,t}^j \\ &\quad + \beta_3 outstanding_{n,t} + \sum_k \gamma_2^k Drdiff_{n,t}^k + maturity_n + \zeta_t + \epsilon_{n,t} \end{aligned} \tag{5}$$

where $rconcession_{n,t}$ is the rate concession between the order placement and execution of security n on day t, and $Drdiff_{n,t}^j$ includes four dummy variables, $d_{10 \leq rdiff < 15}$, $d_{5 \leq rdiff < 10}$, $d_{0 \leq rdiff < 5}$, and $d_{rdiff < 0}$, where each dummy variable equals one when the difference between the principled SLF rate and the SC rate upon order entry ranges between the dummy's subscript values. We expect smaller coefficients for dummies $d_{rdiff < 0}$ or $d_{0 \leq rdiff < 5}$ compared to those for other dummies, because traders are expected to concede little when the SC rate is around the SLF rate. Other regressors are the BoJ's holding rate, $h_{n,t}$; the traded amount in the repo market, $tradeda_{n,t}$; the variables included in the term $Ages_{n,t}^j$; and the logarithm of the outstanding amount, $outstanding_{n,t}$. The term $Ages_{n,t}^j$ includes $age_{n,t}$, $ontherun_{n,t}$, $exontherun_{n,t}$, and $ctd_{n,t}$. Our model includes maturity-level fixed effects $maturity_n$ and time dummies ζ_t to control for maturity- and date-specific effects, respectively. The term $\epsilon_{n,t}$ is the error term. We again restrict our sample to bid orders, in the aim of revealing the impact on short selling activities. We perform the regressions for the whole sample period and for the periods split by the date of relaxation.

[Table 10 about here.]

Table 10 reports the results of the panel regressions of the central bank's lending facility on the SC rate concession. All of the coefficient estimates for the dummies of *rdiff* are significantly positive for the period before the relaxation. The estimates are 0.6809 and 0.6411 for $d_{10 \leq rdiff < 15}$ and $d_{5 \leq rdiff < 10}$, respectively, and they decline to 0.4898 for $d_{0 \leq rdiff < 5}$ and further decline to 0.1751 for $d_{rdiff < 0}$, where bond borrowers concede little. We have similar results for the period after the relaxation. The rate concession is the greatest (0.8120 bps) for *rdiff* between 5 bps and 10 bps. Bond borrowers concede less (0.2634 bps) when the rate difference is below 5 bps, and the coefficient is not significant for bonds with a negative difference. The results indicate the rate concession is smaller when the ordered SC rate is around the SLF rate. These results are consistent with Hypothesis 3: the BoJ's lending facility has a ceiling effect.

We find a ceiling effect on the rate concession for the periods both before and after the relaxation. The concession amount of bidders diminishes when the SC rate approaches the SLF rate. The bidders in the repo market order and modify their rates taking care not to exceed the SLF rate. The SLF lending rate set by the central bank has a great influence on borrower behavior. Among previous studies, Arrata et al. (2020) studies the impact of the ECB's purchases on SC repo rate and show that the special collateral (SC) rate drops by 0.78 bps when 1% of the outstanding amount of government bonds is purchased. They also show the positive effect on the SC rate (0.59-0.76 bps higher rate) during the period of implementation of the SLF.[26] Kinugasa and Nagano (2017) shows that the SLF reduces scarcity premiums and the specialness declines by about 2.7 bps for bonds lent through the SLF.[27] We quantify friction with more concrete measures. Consistent to the previous studies, our empirical results indicate that the SLF lending mitigates the search frictions in the repo market which is increased by the central bank LSAPs.

7 Concluding remarks

In this study, we investigate the impact of a central bank's two monetary policies, LSAPs and the SLF, on the government bond market. These two monetary policies have opposite effects on liquidity. When a central bank implements LSAPs, it reduces the flow of bonds. When bond scarcity is high as a result of LSAPs, a bond dealer who holds a short position will face difficulty finding counterparties. Facing increased search friction, a central bank facilitates

[26] Under the SLF, the ECB lends a specific security against cash at a rate equal to the ECB deposit facility rate minus 30 bps.

[27] They analyze the period before relaxation of the SLF rate.

the SLF to mitigate the shortage of the bond supply. We incorporate the interactions among borrowers, lenders, and a central bank's lending facility into a search-theoretic model and solve the utility functions for the repo lending fee. According to our search-theoretic model, an increase in short sellers (borrowers) caused by LSAPs tightens the demand–supply situation, but the SLF can mitigate a tight demand–supply situation. Our model predicts that the difference between the rates in the repo market and those of the SLF affects dealers' choice and that the central bank's SLF has a ceiling effect on the SC repo rate.

Our empirical analyses find that central bank purchase operations increase repo bid orders. The results of threshold dummy variable analysis indicate that the sensitivity of order imbalance is significantly different when the amounts purchased by the BoJ exceed ¥9 billion, equivalent to 1.4% of the outstanding. The SLF gives dealers a choice between the repo market and the SLF to cover their short positions. The rate difference between the two has a crucial effect on their choice. We find that the proportion for SLF lending increases for bonds whose repo rate increases above the SLF rate. Our results suggest that the repo rates are sensitive to the fees the central bank imposes for its lending facility. We find a ceiling effect on the lending rate in the repo market, and the panel probit regression shows the likelihood of SLF lending depends on the rate difference between the repo market and the SLF. Using order-/execution-level data from the JBOND repo platform, we estimate the bargaining power of lenders and borrowers. Our results indicate that, when bond scarcity is high, the price bargaining power of lenders is stronger than that of the borrowers, and this power balance is affected by the SLF lending rate set by the BoJ. The results are consistent with our model prediction.

The overall results in our four years sample period indicate the followings: The BoJ reduced aggressiveness of the LSAPs around the fourth year of the period. It mitigates the order imbalance in the repo market significantly and raises SC rate.[28] In the middle of the fourth year, the BoJ revised the SLF rate. If this revision of the rate were too large, it had led to generate greater amount of lending from the SLF. But it did not happen and stayed limited amount similar to the amount before the revision. We believe that revision of the SLF rate was apt to the market condition and set the ceiling for lending rates. The SLF has effects of countercyclical policy on liquidity.

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[28] As shown in Table 1, the BoJ reduced its purchases of 5-year and 10-year bonds since 2018. Along with the decrease in the amount purchased, the order imbalance decreased by 2.7%, the average SC rate rose from −15.3 bps to −11.8 bps, and the specialness decreased from 6.5 bps to 3.0 bps between 2017 and 2019.

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A Search-theoretic model

In this section, we construct a model based on a search-theoretic model. We refer to the models of Duffie et al. (2005) and Ferdinandusse et al. (2020) to clarify the impact of increasing central bank holdings and the cost of the SLF on the repo market.

We consider an infinite-horizon steady-state economy. We assume three types of investors in the bond lending market: borrowers, lenders, and a central bank. Borrowers, whose measure is α_{bo}, are short sellers who gain profits by purchasing bonds at low prices and selling them at high prices. They have already agreed to sell a transaction asset of value one to the central bank or other investors. Since they do not hold the bond at the time they decide to sell, they try to borrow the bond either in the repo market or through the SLF run by the central bank. Borrowers, however, are not always short sellers, and we assume, for the sake of simplicity, no other borrowers in the model. The lenders, whose measure is α_l, are non–central bank bondholders. They each hold a unit quantity of bonds and do not sell them, making a profit by lending their bonds in the repo market. The ratio $\frac{\alpha_{bo}}{\alpha_l}$ is the ratio of supply to demand in the repo market and represents the tightness of the repo market. The higher the ratio, the harder it is to find a counterparty in the repo market. We now introduce the central bank not only as a bondholder but also as a lender, which is an aspect not covered in previous work. The central bank holds the bonds it purchases to maturity and lends them through the SLF. Since the central bank's SLF rate is set lower than the SC repo rate in most cases, borrowers borrow bonds through the SLF only when they cannot find a repo counterparty with a better rate than that through the SLF.

We assume that, in the steady state, the lifetime utility of a lender or borrower is the present value of their expected utility flow, net of payments for asset transactions, discounted at a rate $r > 0$. We define V_l, V_{bo}, and V_s as the utilities of lenders, borrowers (short sellers), and impatient bondholders, respectively. Since we focus only on the bond lending market, we consider the bond spot market exogenously and define these utilities obtained through lending or borrowing assets. Therefore, utilities such as those of impatient bondholders who exit the bond lending market are set to zero in our model.

We assume lenders need to pay a small holding cost e_l in each period. We also assume that the probability of finding a counterparty depends on the number of such counterparties in the market. A lender finds a borrower (short seller) with probability $\lambda\alpha_{bo}$ and obtains ω for the bond in a successful repo transaction. Note that λ is the Poisson arrival intensity, and lenders and borrowers are matched with intensity λ. That is, given a group of borrowers with mass α_{bo}, a particular lender meets a borrower with probability $\lambda\alpha_{bo}$. In our model, lenders can switch from a patient to an impatient agent with probability κ, seeking to sell the asset. A borrower pays a positive lending fee ω to the lender. Specialness is calculated by dividing ω by the bond price, and the implied SC rate is the difference between the risk-free rate r and the specialness. If a lender cannot find a counterparty in the repo market, the lender will not make a profit and will need to pay the holding cost in the period. The lenders' utility V_l is thus

$$V_l = \frac{1}{1+r}\left(-e_l + \lambda\alpha_{bo}\omega + \kappa V_s + (1 - \lambda\alpha_{bo} - \kappa)V_l\right) \tag{6}$$

We next consider the utility of borrowers (short sellers). When a borrower borrows a bond to deliver to a buyer, we assume that the borrower borrows the bond in the repo market with probability β and borrows through the SLF with probability $1-\beta$. When a bond is not scarce and there is a sufficient bond supply in the repo market, the borrowing fee can be kept low and more favorable than that of the SLF. In this case, most lending transactions are carried out in the repo market, that is, β is close to one. On the other hand, if the rate of the repo market and the SLF rate are at the same level, borrowing in the repo market and through the SLF will be equivalent for borrowers, so β will be close to 0.5.

In our model, we consider a short seller makes a profit by purchasing bonds at low prices and selling them at higher prices in the JGB spot market. We define h_s as the profit arising from the sales and purchases of a bond. Second, we define the borrowing cost. If a borrower is not having a difficult time finding a lender in the repo market, the borrower will try to borrow the bond to be sold in the repo market (with probability β), rather than borrowing from the central bank. In the repo market, the borrower finds a lender with probability $\lambda\alpha_l$, incurs a cost ω for the borrowing fee when the repo transaction succeeds, and then exits the repo market. The term λ is the same Poisson intensity as in Eq. (6). When a borrower cannot find a lender offering a more favorable rate than the SLF rate, the borrower will then try to borrow the bond from the central bank. We define δ as the probability of the borrower being successful at borrowing through the central bank's SLF. The term δ is assumed to remain constant for LSAPs, because a central bank will try to respond to investors' demand for bond lending to maintain the liquidity of the JGB market, which tends to decline due to LSAPs. Let the central bank's lending fee be ω^{cb}, which is set much higher than ω in most cases. When a borrower can borrow the bond in the repo market or through the SLF, the borrower exits the bond lending market and the borrower's profit is the difference between the profit h_s and the borrowing cost ω (or ω^{cb}). We also consider the case in which a short seller (borrower) cannot cover the short position. If the short seller does not hold the bond on the sale date, the short seller's transaction will fail in the bond spot market. Let q be the probability of failure and $h_f(> 0)$ be the cost of failure, which includes penalties such as a decline in the short seller's credit. Considering these cases together, we find that the utility of borrowers is

$$V_{bo} = \frac{1}{1+r}\left(\beta\lambda\alpha_l(h_s - \omega) + (1-\beta)\delta(h_s - \omega^{cb}) - qh_f + (1 - \beta\lambda\alpha_l - (1-\beta)\delta - q)V_{bo}\right) \tag{7}$$

We now determine the lending fee by introducing bargaining power. When a borrower finds a lender in the repo market, the two bargain bilaterally over the lending fee. We assume the lending fee is determined through Nash bargaining, which is applied by Duffie et al. (2005) and Ferdinandusse et al. (2020), among others. Nash bargaining implies that the bargaining process results in the lending fee

$$\omega = \phi V_l + (1-\phi)\left(h_s - V_{bo}\right) \tag{8}$$

for some $\phi \in [0,\ 1]$, where ϕ measures the borrower's bargaining power.

We then solve the lending fees with Eqs. (6) to (8) as follows:

$$\begin{aligned}\omega \quad = \quad & \frac{(\delta-\beta\delta+\beta\lambda\alpha_l)h_s - qh_f - (1-\beta)\delta\omega^{cb}}{\beta\lambda\alpha_l} \\ & + \frac{(\beta\lambda\alpha_l(h_s(1-\phi)F - e_l\phi))\, G}{\beta\lambda\alpha_l\left((r+\kappa+\lambda\alpha_{bo}(1-\phi)G - \beta\lambda\alpha_l(1-\phi)F\right)} \\ & - \frac{\left((r+\kappa+\lambda\alpha_{bo}(1-\phi))(-qh_f + (\delta-\beta\delta+\beta\lambda\alpha_l)h_s + (1-\beta)\delta\omega^{cb})\right) G}{\beta\lambda\alpha_l\left((r+\kappa+\lambda\alpha_{bo}(1-\phi)G - \beta\lambda\alpha_l(1-\phi)F\right)}\end{aligned} \tag{9}$$

where

$$F(r,\kappa,\lambda,\alpha_{bo}) \quad = \quad r+\kappa+\lambda\alpha_{bo} \tag{10}$$

$$G(r,q,\beta,\delta,\lambda,\alpha_l) \quad = \quad r+\beta\lambda\alpha_l + (1-\beta)\delta + q \tag{11}$$

Figure 1: Operation timeline

This figure shows the timeline of a repo transaction, a central bank purchase operation, and lending through the SLF.

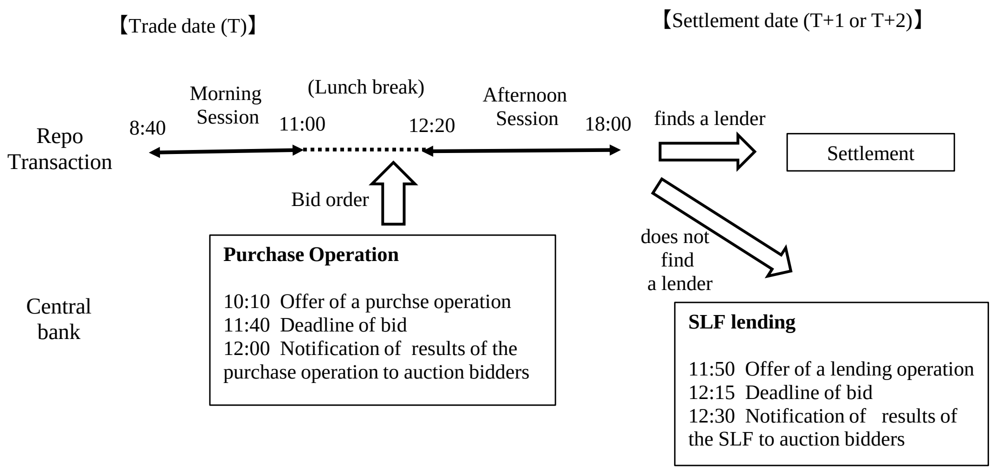

Figure 2: Historical SLF lending rate

This figure shows the principled SLF lending rate (in bps, left axis), the average SLF lending rate (in bps, left axis), and the amount lent through the SLF in a day (in trillions of yen, right axis). The principled SLF lending rate is calculated as the uncollateralized overnight call rate minus 50 bps for the period before the relaxation and the Tokyo Repo Rate minus 25 bps for the period after the relaxation. The dotted vertical line is the date of the relaxation of conditions for the SLF. The data cover the period from April 1, 2016, to December 31, 2019.

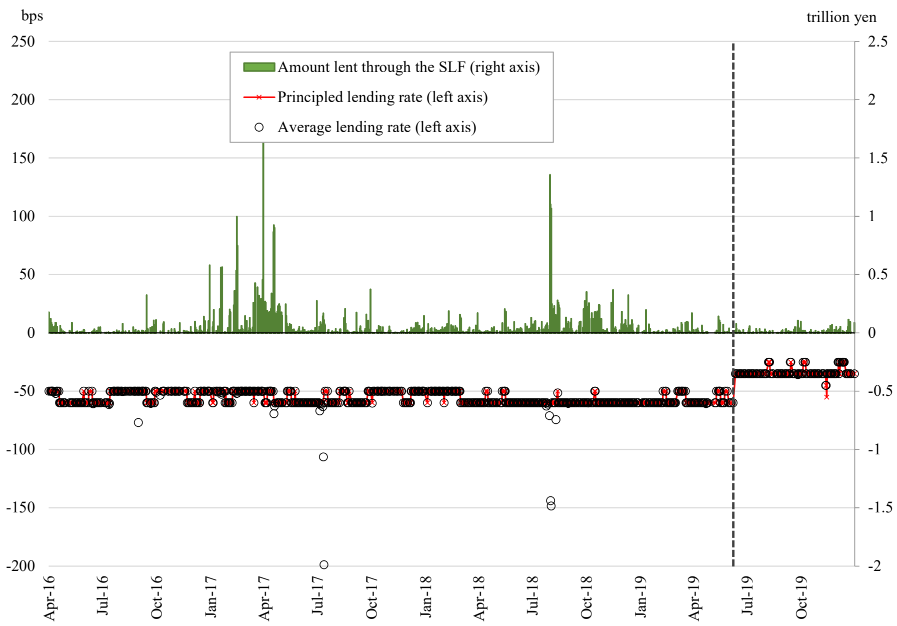

Figure 3: Historical uncollateralized overnight call rate and repo rate
This figure shows the time series evolution of the SC rate in bps (blue crosses), the Tokyo Repo Rate in bps (yellow circles), and the uncollateralized overnight call rate in bps (black dashed line). The uncollateralized overnight call rate is obtained from the BoJ, and the Tokyo Repo Rate is obtained from the Japan Securities Dealers Association. The data cover the period from April 1, 2016, to December 31, 2019. The dotted vertical line is the date the conditions are relaxed for the SLF.

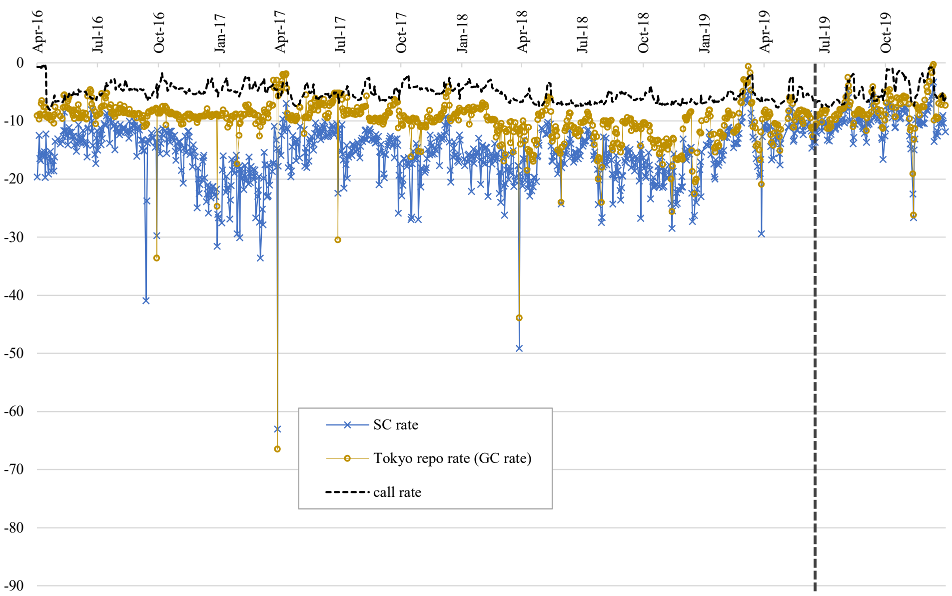

Figure 4: Contribution of purchase operations and scarcity to order imbalance
This figure shows the calculated contribution of purchase operation variables to order imbalance based on model (M1) in Table 5. The contribution is calculated as $\hat{\beta}_1 h_{n,t} + \hat{\beta}_2 h_{n,t}^2 + \hat{\gamma}_1 logpo_{n,t} + \hat{\gamma}_2 logpo_{n,t} h_{n,t}$, where $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are coefficient estimates of (M1). We calculate these for BoJ holding rates $h_{n,t}$ of 0%, 20%, 40%, 60%, and 80%. The sample comprises five- and 10-year bonds traded on the repo market.

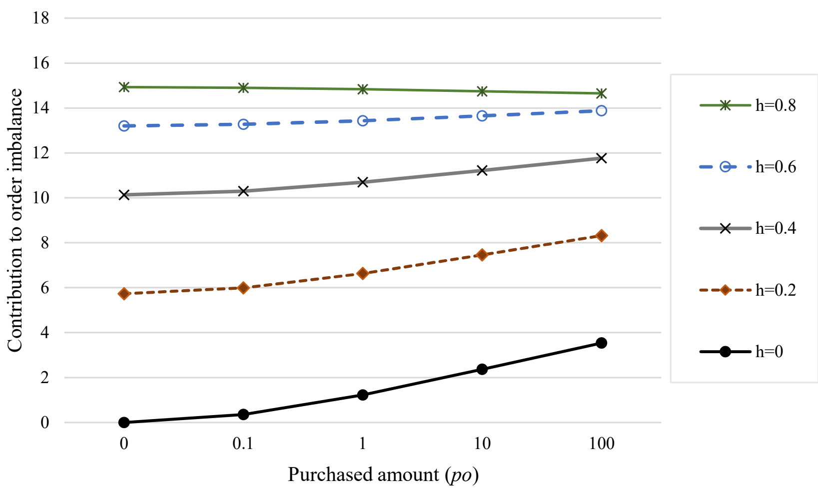

Figure 5: Results of the threshold dummy variable analysis
The dependent variable is the order imbalance (%) and the regression is presented in Eq. (3). The dotted line denotes the sequential p-values of the t significance test of β_3 for different thresholds χ. The horizontal line shows the threshold of 5%. The solid line shows the coefficient estimates of the threshold dummy variable $dpo(\chi)h_{n,t}$ for different χ values. Those values that are significant at the 5% level are displayed with a thicker line. The test is executed only when the dummy subsample contains over 5% of the whole sample.

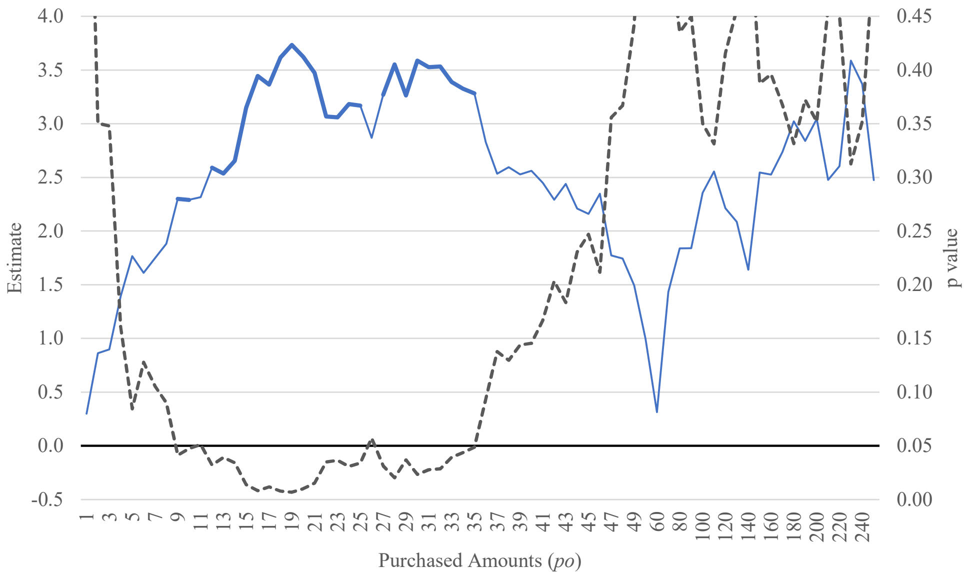

Table 1. Summary of the BoJ's purchase operation

This table shows the average monthly purchase amounts and amounts purchased per security per purchase operation. Panel A shows the average monthly amounts purchased by the BoJ (in billions of yen) and its end-of-month holding rate (as a percentage) averaged over the year. Panel B shows the statistics of the BoJ's purchased amounts (in billions of yen) per security per operation. The sample comprises all five- and 10-year original maturity bonds outstanding.

Panel A: Monthly purchased amounts and the BoJ's holding rate

	Purchased Amount		Holding Rate
Year	5-Year Bond	10-Year Bond	%
2016	2885.3	2700.8	45.6
2017	2253.8	2761.3	53.8
2018	1836.7	2647.8	60.1
2019	1618.7	1878.1	61.9
ALL	2099.5	2483.4	56.0

Panel B: Purchased amounts per security per operation

Year	Mean	Min	1st Q	Median	3rd Q	Max
2016	37.4	0.03	1.2	5.1	23.7	447.6
2017	50.1	0.03	1.1	5.0	22.1	446.8
2018	45.7	0.03	1.4	6.5	29.4	450.7
2019	35.8	0.05	1.1	5.9	25.2	448.5
ALL	42.0	0.03	1.1	5.6	25.0	450.7

Table 2. Parameter values used in the calibration

Parameters	Definition	Before Relaxation	After Relaxation
$1/\kappa$	Investment horizon (years)	0.77	
q	Probability of failure	0.0080	
δ	Probability of successful bid of the SLF	0.999	
λ	Poisson intensity of the search process	100000	
h_s	Profit from sales and purchases	0.00036	
h_f	Cost of failure	0.03	
e_l	Lenders' search cost	0.00001	
r	Risk-free rate	-0.00035	
ϕ	Lenders' bargaining power	0.5	
ω^{cb}	Central bank's lending fee	0.005	0.0035
η_1	Parameter of error function	550	750

Table 3. Calibration results

This table shows the values of the repo lending fees, ω, and the selection probability of the repo transaction, β, arising from model calibration under the parameters for the periods before and after the relaxation date of the SLF conditions, June 10, 2019. Column (1) shows the results for the case in which the central bank does not lend bonds through the SLF. Column (2) shows the results for the case in which the central bank lends bonds from its holdings, based on the parameters associated with the period before relaxation in Table 2. Column (3) shows the results based on the parameters associated with the period after relaxation. The terms ω and β are shown for different measures of lenders, α_l, which is the indicator of scarcity. Duration time is determined based on the α_l value.

		(1)	(2)		(3)	
		Without SLF	With SLF			
Scarcity			Before Relaxation		After Relaxation	
α_l	Duration Time	ω	ω	β	ω	β
0.1	12	9.07	9.19	(99.92%)	9.42	(99.67%)
0.05	24	17.70	20.06	(99.01%)	24.72	(93.65%)
0.01	120	72.53	52.52	(42.24%)	37.85	(38.11%)

Table 4. Scarcity and order imbalance

This table presents the average order imbalance for each BoJ holding rate category, the differences in the order imbalances for the two holding rate categories, and the significance of the tests. Order imbalance (%) is calculated as the proportion of bid orders among all orders. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. The sample comprises all five- and 10-year original maturity bonds traded in the repo market.

Holding Rate	Average Imbalance	Welch Test				Observations
		$0 \leq h < 0.2$	$0.2 \leq h < 0.4$	$0.4 \leq h < 0.6$	$0.6 \leq h < 0.8$	
$0 \leq h < 0.2$	45.6					4154
$0.2 \leq h < 0.4$	46.9	1.3 **				15879
$0.4 \leq h < 0.6$	49.1	3.5 ***	2.2 ***			12234
$0.6 \leq h < 0.8$	53.2	7.6 ***	6.3 ***	4.1 ***		18855
$0.8 \leq h < 1$	57.9	12.3 ***	11.0 ***	8.8 ***	4.7 ***	9662

Table 5. Impact of purchase operations by the central bank on order imbalance
This table presents the results for the regression investigating the impact of the central bank's purchase operations on order imbalance. The dependent variable is the order imbalance (%), and the model is described in Eq. (1). Original security-level fixed effects and daily time dummies are not shown. The sample comprises five- and 10-year bonds traded in the repo market in a day. The t-values are in parentheses and are calculated with cluster-robust standard errors. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Regression of order imbalance

	(M1)	(M2)
h	0.3202 ***	0.6355 ***
	(5.12)	(8.42)
h^2	−0.1669 ***	−0.2648 ***
	(−2.88)	(−3.92)
$logpo$	0.0051 ***	0.0049 ***
	(2.91)	(2.78)
$logpo \times h$	−0.0069 **	−0.0067 **
	(−2.35)	(−2.28)
$ontherun$		0.0377 **
		(2.53)
$exontherun$		0.0195 **
		(2.35)
age		−40.6940
		(−1.03)
ctd		0.0242 **
		(2.53)
$outstanding$		−0.1280 ***
		(−7.23)
Time fixed	YES	YES
Security fixed	YES	YES
Observations	60784	60784
Adjusted R-squared	0.14053	0.14049

Table 6. Impact of relaxation on repo transactions

This table shows the number of bid orders per day for each SC rate level and the proportion for each rate difference between the principled SLF rate and the SC repo rate. Panel A presents the number of bid orders per day for each SC rate level and its proportion. Panel B presents the proportion of bid orders for each rate difference between the principled SLF rate and the SC repo rate. We split our sample period by the date of relaxation. The sample comprises the repo transactions of five- and 10-year original maturity bonds.

Panel A: Number of bid orders for each SC rate level and its proportion

	$-10 < sc$	$-20 < sc \leq -10$	$-30 < sc \leq -20$	$-40 < sc \leq -30$	$-50 < sc \leq -40$	$-60 < sc \leq -50$	$sc \leq -60$	ALL
Before	78.0	316.3	71.1	29.1	22.0	13.3	10.1	540.0
Relaxation	(14.4%)	(58.6%)	(13.2%)	(5.4%)	(4.1%)	(2.5%)	(1.9%)	
After	272.3	245.6	17.3	7.5	0.1	0.0	0.0	542.8
Relaxation	(50.2%)	(45.2%)	(3.2%)	(1.4%)	(0.0%)	(0.0%)	(0.0%)	

Panel B: Proportions (%) of orders for each rate difference between the SLF rate and the SC rate

	$25 \leq rdiff$	$20 \leq rdiff < 25$	$15 \leq rdiff < 20$	$10 \leq rdiff < 15$	$5 \leq rdiff < 10$	$0 \leq rdiff < 5$	$rdiff < 0$
Before Relaxation	88.2%	2.7%	1.9%	2.0%	1.4%	1.9%	2.0%
After Relaxation	51.2%	38.4%	7.2%	1.5%	0.8%	0.7%	0.3%

Table 7. Panel probit analysis of the SLF

This table presents the results for the panel probit regression of the probability of non-execution. Panel A shows the results for the period before the relaxation and Panel B for that after the relaxation. The dependent variable is a binary variable that equals one if and only if the bond is lent through the SLF, and the explanatory variables are presented in Eq. (4). The z-values are in parentheses. The marginal effects (MEs) for *sc* and *rdiff* are calculated as the average for each SC rate or rate difference level. The sample comprises all five- and 10-year bonds that are traded in the repo market. The subscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Before Relaxation

	(A1)	(A2)	(A3)	(A4)	(A5)
sc	−0.0353 ***	−0.0488 ***			
	(−44.4)	(−33.9)			
rdiff			−0.0333 ***	−0.0488 ***	
			(−42.6)	(−33.9)	
h					4.003 ***
					(13.0)
imb					0.1950 ***
					(3.86)
logpo					0.0035
					(0.52)
ontherun					1.4620 ***
					(12.8)
exontherun					0.7580 ***
					(12.0)
age					99.70
					(0.23)
ctd					0.6655 ***
					(9.76)
outstanding					−0.8878 ***
					(−7.00)
Time fixed	NO	YES	NO	YES	YES
Security fixed	NO	YES	NO	YES	YES
Observations	51406	51406	51406	51406	51406
Log-likelihood	2085.3 ***	4336.0 ***	1979.3 ***	4336.0 ***	3463.3 ***
Pseudo-R-squared	0.1373	0.2855	0.1303	0.2855	0.2280
ME ($-10 < sc$)	−0.0013	−0.0018			
ME ($-20 < sc \leq -10$)	−0.0018	−0.0021			
ME ($-30 < sc \leq -20$)	−0.0037	−0.0040			
ME ($-40 < sc \leq -30$)	−0.0062	−0.0073			
ME ($-50 < sc \leq -40$)	−0.0093	−0.0114			
ME ($sc \leq -50$)	−0.0125	−0.0148			
ME ($50 \leq rdiff$)			−0.0010	−0.0014	
ME ($40 \leq rdiff < 50$)			−0.0015	−0.0021	
ME ($30 \leq rdiff < 40$)			−0.0025	−0.0024	
ME ($20 \leq rdiff < 30$)			−0.0047	−0.0057	
ME ($10 \leq rdiff < 20$)			−0.0073	−0.0098	
ME ($rdiff < 10$)			−0.0100	−0.0131	
ME of *h*					0.2394
ME of *imb*					0.0117
ME of *logpo*					0.0002
ME of *ontherun*					0.0874
ME of *exontherun*					0.0453
ME of *ctd*					0.0398
ME of *outstanding*					−0.0531

Table 7. Panel probit analysis of the SLF (continued)

Panel B: After Relaxation

	(B1)	(B2)	(B3)	(B4)	(B5)
sc	−0.0459 ***	−0.0753 ***			
	(−9.18)	(−7.21)			
rdiff			−0.0493 ***	−0.0753 ***	
			(−9.69)	(−7.21)	
h					1.822
					(1.57)
imb					−0.1582
					(−1.09)
logpo					0.0391 **
					(2.01)
ontherun					0.9583 **
					(2.18)
exontherun					−0.0187
					(−0.06)
age					−2524 **
					(−2.18)
ctd					−0.0558
					(−0.18)
outstanding					0.0171
					(0.05)
Time fixed	NO	YES	NO	YES	YES
Security fixed	NO	YES	NO	YES	YES
Observations	9378	9378	9378	9378	9378
Log-likelihood	83.4 ***	558.6 ***	92.3 ***	558.6 ***	523.6 ***
Pseudo-R-squared	0.0385	0.2577	0.0426	0.2577	0.2416
ME ($-10 < sc$)	−0.0019	−0.0031			
ME ($-20 < sc \leq -10$)	−0.0029	−0.0031			
ME ($-30 < sc \leq -20$)	−0.0070	−0.0073			
ME ($-40 < sc \leq -30$)	−0.0115	−0.0207			
ME ($-50 < sc \leq -40$)	–	–			
ME ($sc \leq -50$)	–	–			
ME ($50 \leq rdiff$)			–	–	
ME ($40 \leq rdiff < 50$)			–	–	
ME ($30 \leq rdiff < 40$)			−0.0012	−0.0043	
ME ($20 \leq rdiff < 30$)			−0.0024	−0.0030	
ME ($10 \leq rdiff < 20$)			−0.0045	−0.0041	
ME ($rdiff < 10$)			−0.0103	−0.0136	
ME of *h*					0.0832
ME of *imb*					−0.0072
ME of *logpo*					0.0018
ME of *ontherun*					0.0438
ME of *exontherun*					−0.0009
ME of *ctd*					−0.0025
ME of *outstanding*					0.0008

Table 8. Order duration and scarcity

This table shows the average duration for each BoJ holding rate category and the results of Welch two-sample t-tests. The duration (in minutes) is calculated as the time from the bid order placement until execution. The average duration for each BoJ holding rate category is shown in the table. The sample is split into two by the date of the relaxation of the SLF conditions (June 10, 2019). In the diff column, the differences between the average duration time before relaxation and after relaxation are shown. The Welch two-sample t-test's alternative hypothesis is that the true difference in means is not equal to zero. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. The sample comprises all bid orders for five- and 10-year original maturity bonds filled in the repo market.

	Before Relaxation	After Relaxation	Diff
$0 \leq h < 0.2$	6.23	6.05	−0.18
$0.2 \leq h < 0.4$	5.50	5.17	−0.33 *
$0.4 \leq h < 0.6$	6.09	7.90	1.80 ***
$0.6 \leq h < 0.8$	10.05	13.63	3.58 ***
$0.8 \leq h < 1$	11.53	14.92	3.39 ***

Table 9. Bargaining power and rate concession

This table shows the rate concession of bid (offer) orders and the proxy for bargaining power. The rate concession (in bps) is calculated as the difference between the initial and final specialness of orders. Our proxy for the bargaining power of borrowers is defined as the ratios of the rate concession amounts for offer orders to the aggregate rate concession amounts for bid and offer orders. The significance of the Welch two-sample t-test, whose alternative hypothesis is that the true difference between the rate concession for bid orders and that for offer orders in means is not equal to zero—that is, the true difference between bargaining power of lenders and borrowers—is indicated by asterisks. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. The sample comprises repo transactions of five- and 10-year original maturity bonds.

	Before Relaxation			After Relaxation		
	Rate Concession		Bargaining	Rate Concession		Bargaining
	Bid	Offer	Power	Bid	Offer	Power
$0 \leq h < 0.2$	0.134	0.152	0.530	0.056	0.075	0.572
$0.2 \leq h < 0.4$	0.117	0.117	0.499	0.046	0.046	0.501
$0.4 \leq h < 0.6$	0.143	0.111	0.438 ***	0.066	0.059	0.475
$0.6 \leq h < 0.8$	0.351	0.284	0.447 ***	0.185	0.201	0.522
$0.8 \leq h < 1$	0.401	0.389	0.492	0.196	0.257	0.567 ***

Table 10. Ceiling effect of the SLF

This table presents the results for the regression investigating the impact of the central bank's lending facility on rate concession. The dependent variable is the rate concession amounts between the initial and final rates (in bps), and the regression equation is presented in Eq. (5). We test whether the central bank's lending facility mitigates the impact on the rate change between the initial and final SC rates. Original maturity-level fixed effects and daily time dummies are not shown. The sample comprises all five- and 10-year bonds that are ordered bids and filled in the repo market. The t-values are in parentheses and are calculated with cluster-robust standard errors. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	ALL	Before Relaxation	After Relaxation
intercept	2.0425 ***	2.2358 ***	0.5391 **
	(7.2)	(7.1)	(1.98)
holding	0.6601 ***	0.7149 ***	0.3635 ***
	(13.8)	(12.3)	(6.91)
tradeda	−0.1538 ***	−0.1655 ***	−0.1071 ***
	(−32.9)	(−30.1)	(−20.14)
ontherun	0.1672 ***	0.1579 ***	0.1481 ***
	(3.85)	(3.11)	(2.76)
exontherun	0.2355 ***	0.2511 ***	0.0604
	(6.61)	(6.09)	(1.35)
age	−0.0105 *	−0.0167 ***	0.0035
	(−2.2)	(−2.8)	(0.73)
ctd	−0.0063	0.0108	−0.0883
	(−0.13)	(0.20)	(−1.60)
outstanding	−0.0731 ***	−0.0879 ***	−0.0106
	(−3.94)	(−4.1)	(−0.44)
$d_{10 \leq rdiff < 15}$	0.6224 ***	0.6809 ***	0.3600 ***
	(10.3)	(9.55)	(5.42)
$d_{5 \leq rdiff < 10}$	0.6659 ***	0.6411 ***	0.8120 ***
	(9.80)	(8.16)	(9.63)
$d_{0 \leq rdiff < 5}$	0.4698 ***	0.4898 ***	0.2634 ***
	(6.58)	(5.98)	(2.79)
$d_{rdiff < 0}$	0.1801 *	0.1751 **	0.0737
	(2.43)	(2.12)	(0.58)
Time fixed	Yes	Yes	Yes
Maturity fixed	Yes	Yes	Yes
Observations	50106	42059	8047
Adjusted R-squared	0.1017	0.100	0.1309